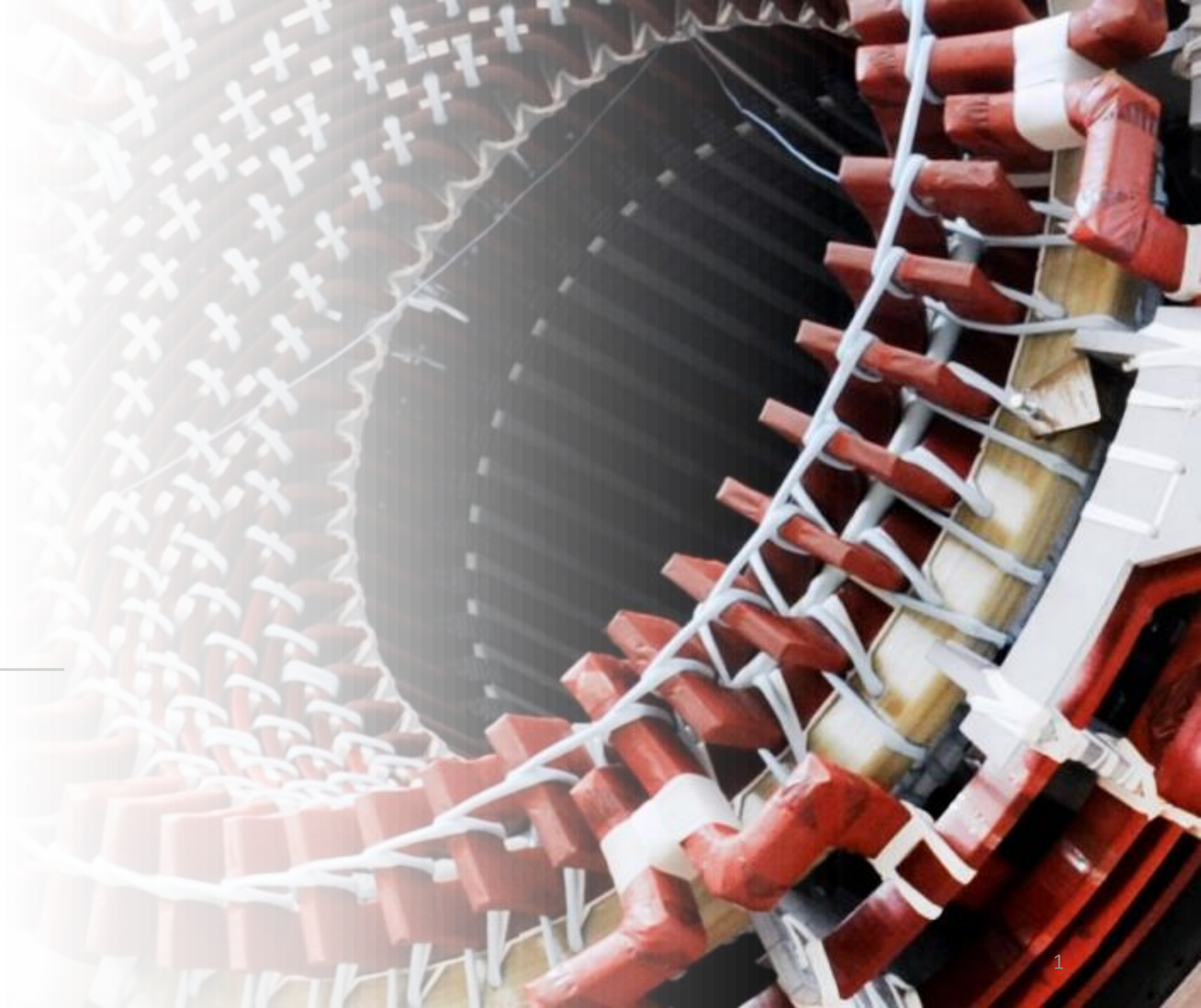
EE3124 Introduction to Electric Machines and Drives

2-Basics of Magnetic Circuit

Prof. CQ Jiang

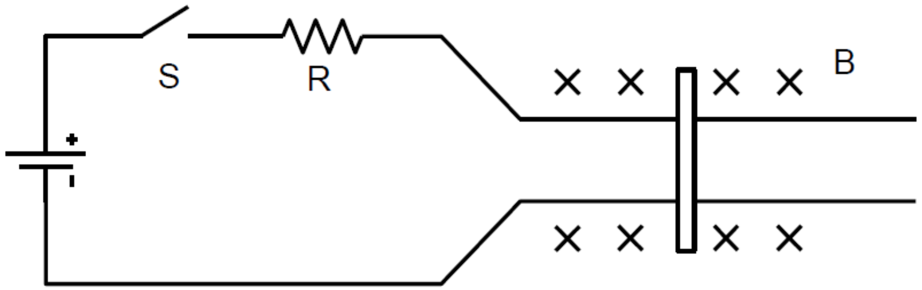


Outline

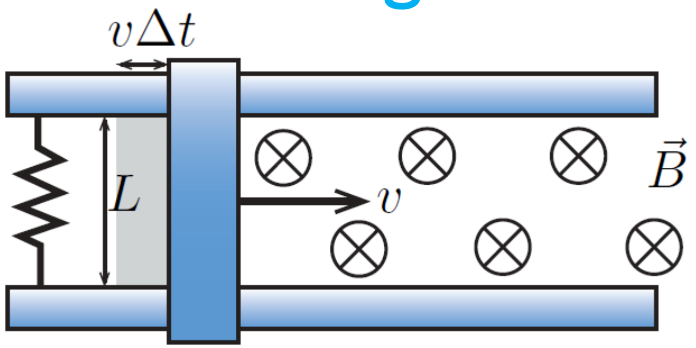
- Ampere's Law
- Faraday's Law of Induction
- Magnetic Flux Density and Permeability
- Relative Permeability - Magnetic Core Characteristics
- Magnetic Circuit – Reluctance
- Flux Linkage and Inductance

Revision of Fleming's Left- or Right-hand Rule

1) Where to go of the bar?



2) What is the voltage across the bar?



Along the bed of this railroad track is a constant, uniform-density magnetic field directed into the page.

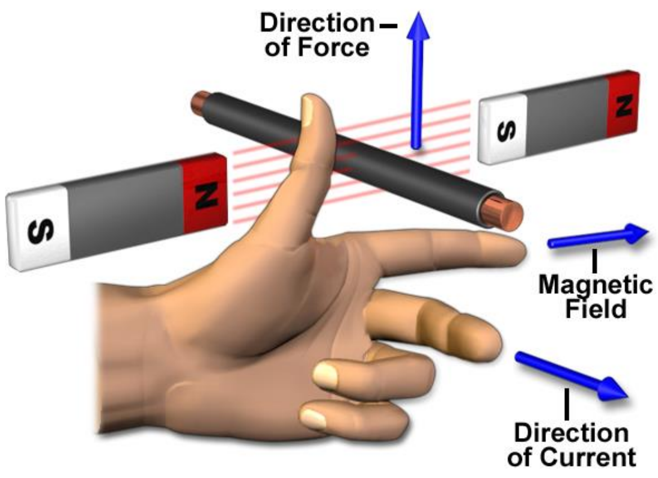
- It consists of a battery and a resistance connected through a switch to a pair of smooth, frictionless rails. A bar of conducting metal is lying across the tracks.
- Close the switch, the bar will accelerate to which direction?

- A bar of conducting metal is moving along the tracks connected to a resistance.
- Along the motion, what is the voltage across the bar?

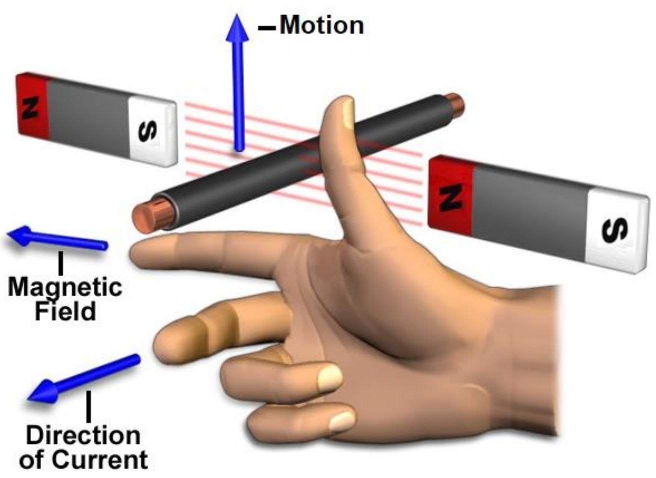
3

Revision of Fleming's Left- or Right-hand Rule

Left-hand Rule



Right-hand Rule



- It is a bit confusing to use either Fleming's left- or right-hand rule.
- Easy to remember

Left hand is more powerful (force) than your right hand!!!

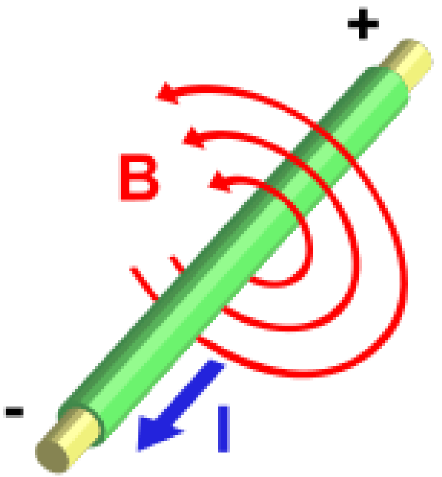
- Left hand rule for creation of force.
- Right hand rule for creation of current or field.

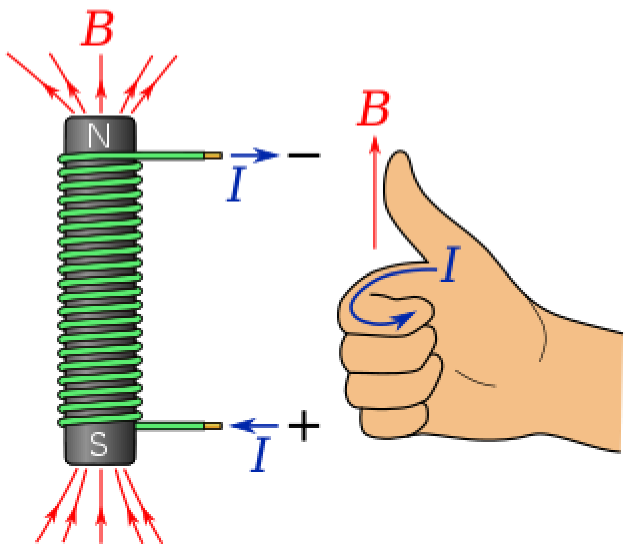
4

Production of a Magnetic Field

- 1820:

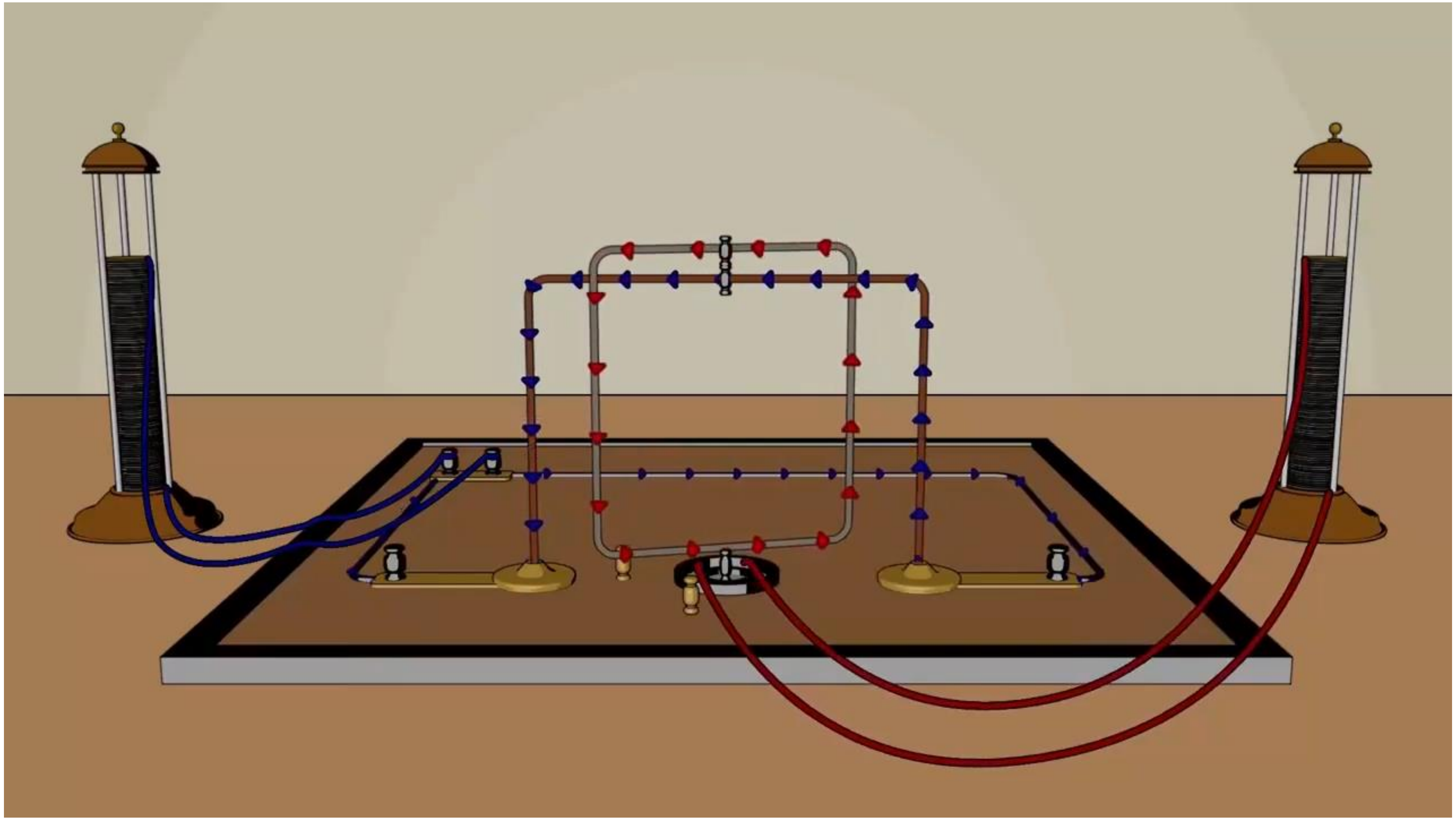
André-Marie Ampère developed Ampere’s law showing that electric current produces a magnetic field. 電流產生磁力場





André-Marie Ampère
French Scientist

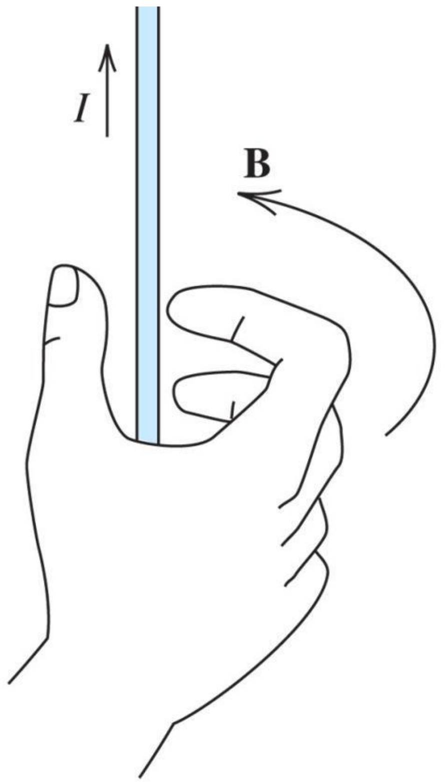
Who is André-Marie Ampère as a French Scientist



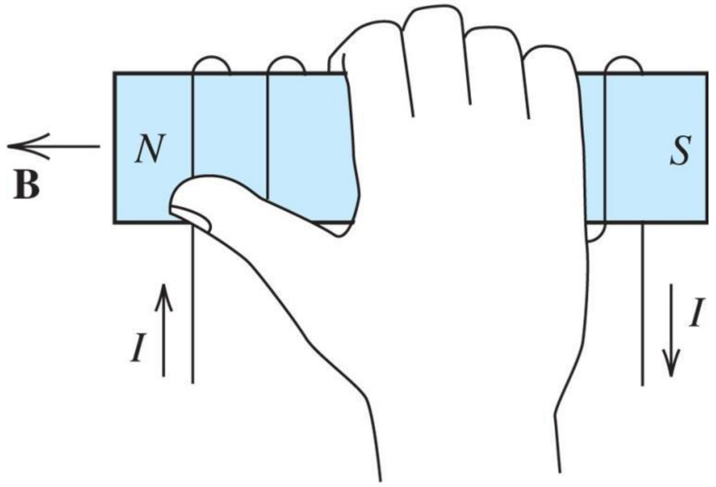
https://www.youtube.com/watch?v=x8BVwosjI1Y&ab_channel=PhilippePonchon

Production of a Magnetic Field

Right Hand Rule



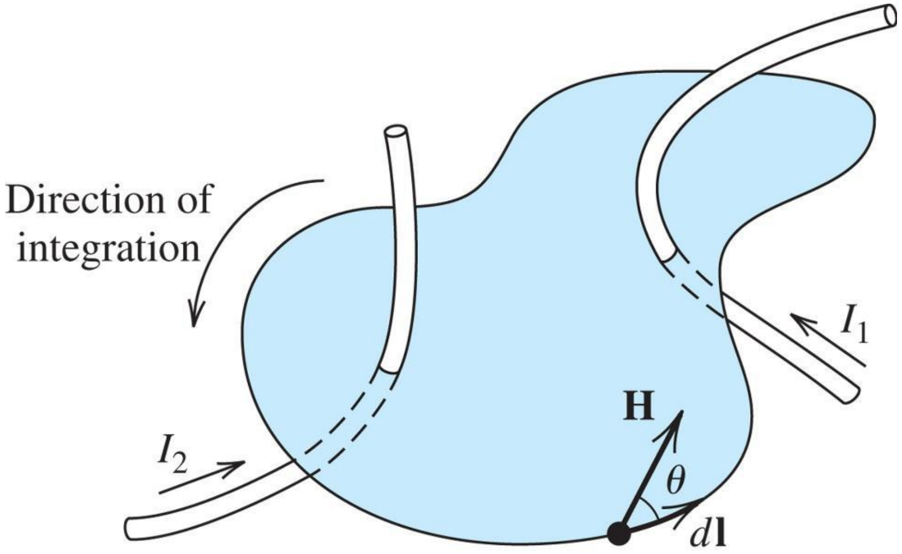
(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field



(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Ampere's Law

- The line integral of the magnetic field around a closed path L is equal to the algebraic sum of the currents flowing through the area enclosed by the path.



$\oint \mathbf{H} \bullet d\mathbf{l} = I_1 + I_2$

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = \sum i}$$

where **H** is the magnetic field intensity produced by the current *i*, *i* is measured in amperes and **H** is measured in ampere-turns per meter.

Ampere's Law

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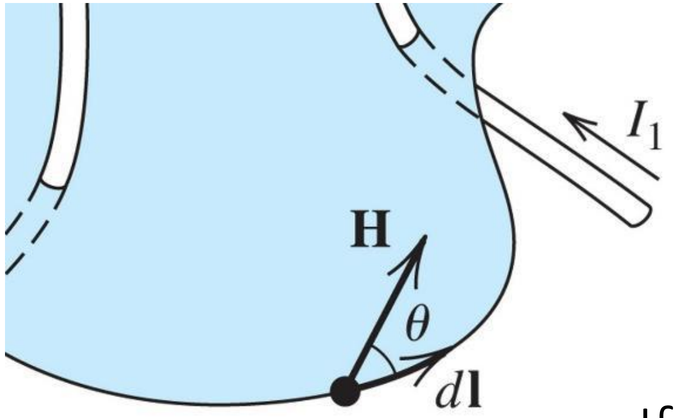


$\oint \mathbf{H} \bullet d\mathbf{l} = I_1 + I_2$

$$\boxed{\oint \mathbf{H} \cdot d\mathbf{l} = \sum i}$$

The vector dot product is:

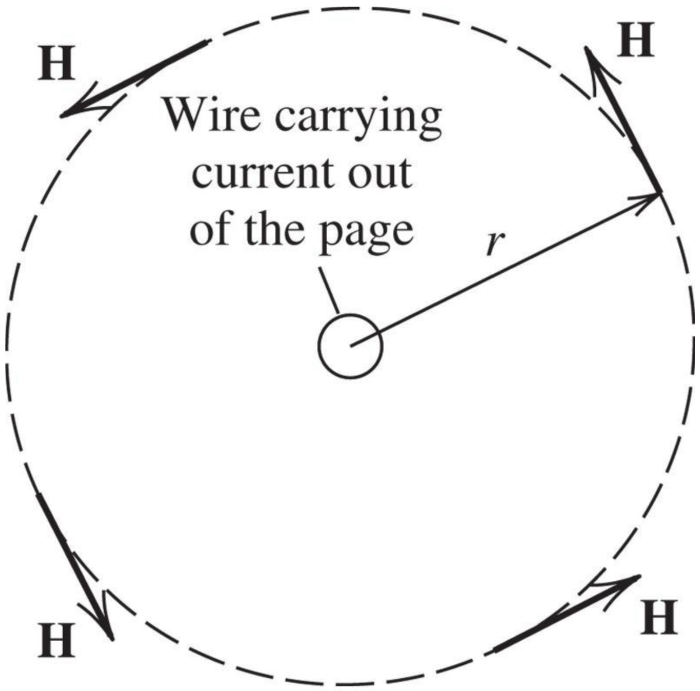
$$\boxed{\mathbf{H} \cdot d\mathbf{l} = H dl \cos(\theta)}$$

If the magnetic field is constant and points in the same direction as *d***l**

$$\boxed{Hl = \sum i}$$

Ampere's Law

- **Example**: A Conductor Carrying a Current



$$Hl = H(2\pi r) = i$$

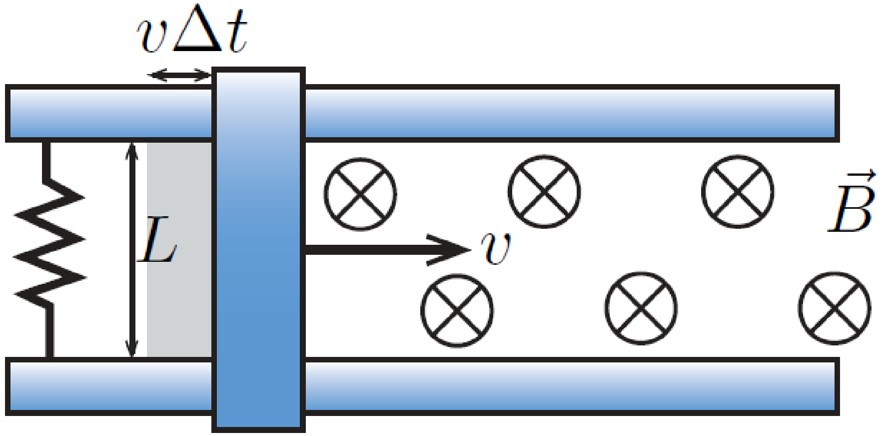
$$H = \frac{i}{2\pi r}$$

Maison D’Ampere in France (Ampère Museum)

Faraday's Law of Induction

- Basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF)—a phenomenon known as electromagnetic induction.
- The electromotive force around a closed path is equal to the negative of the time rate of change of the magnetic flux enclosed by the path.

$$emf = -\frac{d\Phi_{mag}}{dt}$$



In a short time Δt the bar moves a distance Δx = v*Δt, and the flux increases by ΔΦmag = B (L v*Δt)

$$emf = \frac{\Delta\Phi_{mag}}{\Delta t} = BLv$$

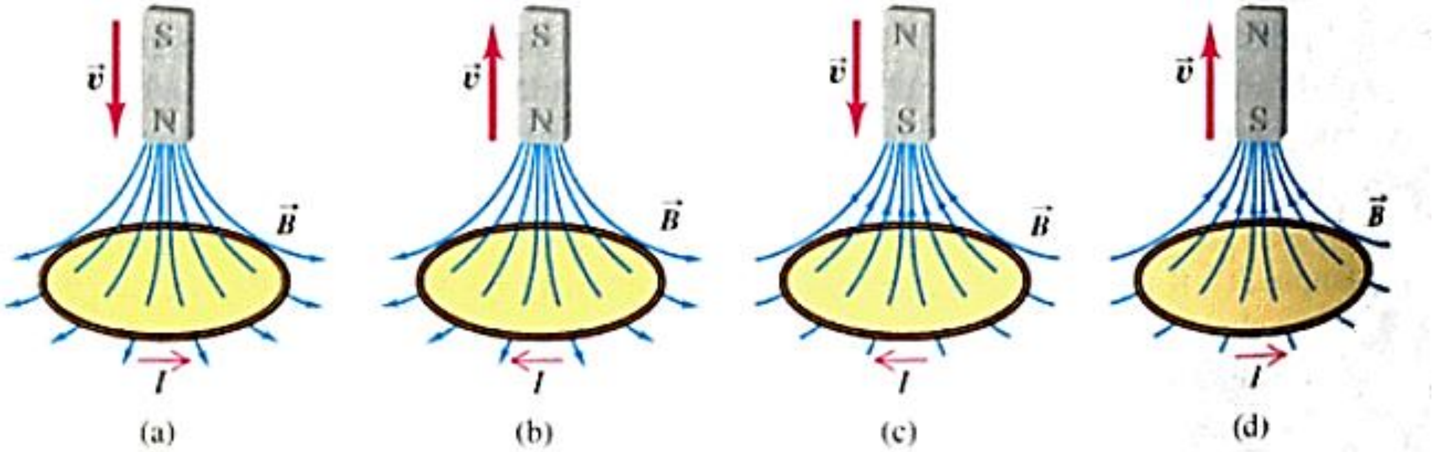
There is an increase in flux through the circuit as the bar of length *L* moves to the right (orthogonal to magnetic field H) at velocity, *v*.

Faraday's Law of Induction

- It indicates that when a magnet moves towards a closing coil, the induced current will be generated (Lenz Law). This simple law governs the basic operation principle of all electric machines.
- No motion, No current.

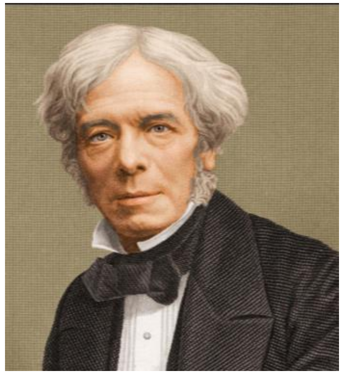
The induced emf in a coil of N turns is equal to N times the rate of change of the magnetic flux on one loop of the coil.

$$emf = -N\frac{d\Phi_{mag}}{dt}$$

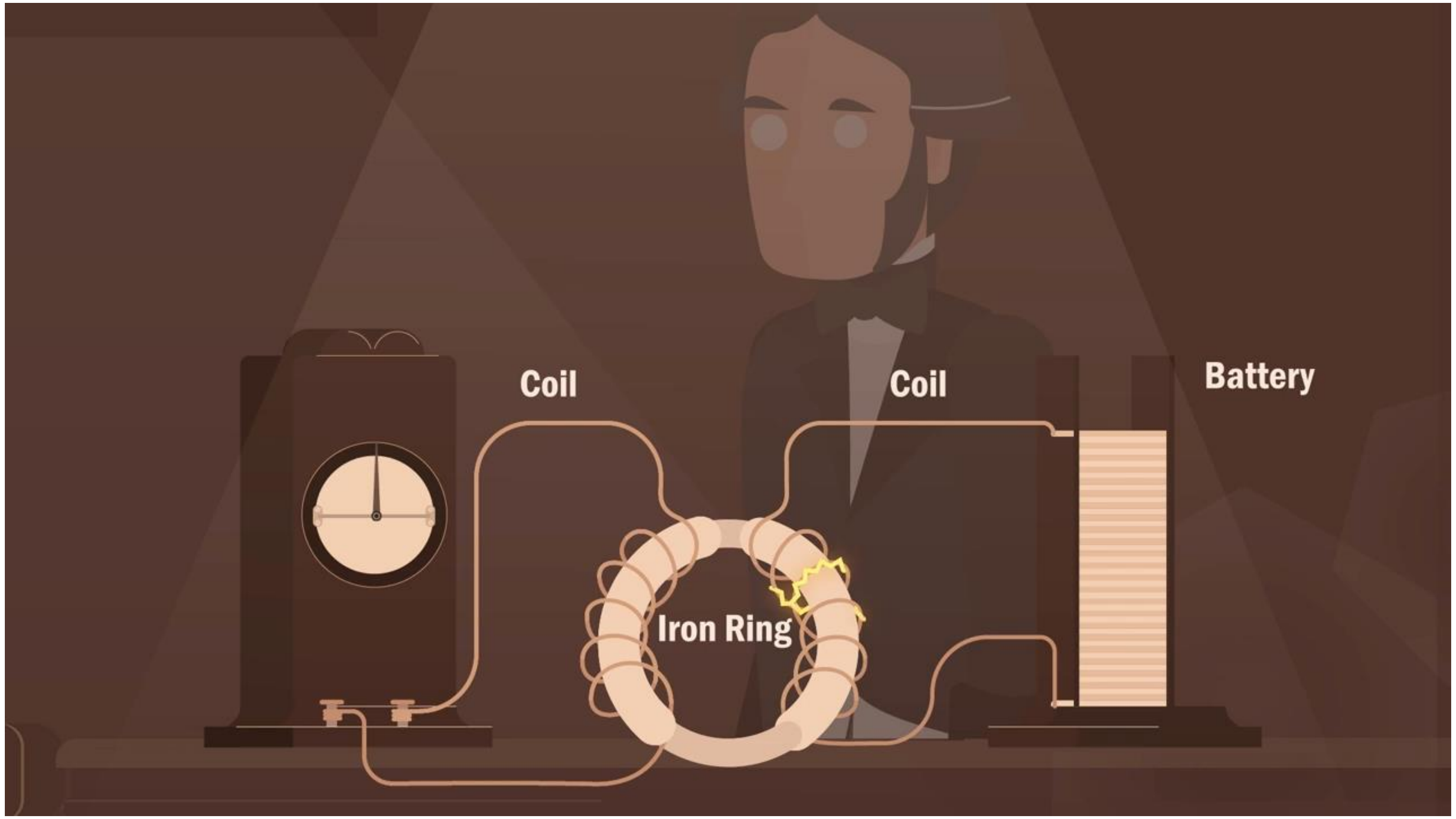


Lenz's law states that:
The current induced in a circuit due to a change in a magnetic field is directed to oppose the change in flux and to exert a mechanical force which opposes the motion.

Who is Michael Faraday



- Michael Faraday FRS (22 September 1791 – 25 August 1867) was an English scientist who contributed to the study of electromagnetism and electrochemistry. His main discoveries include the principles underlying electromagnetic induction, diamagnetism and electrolysis.



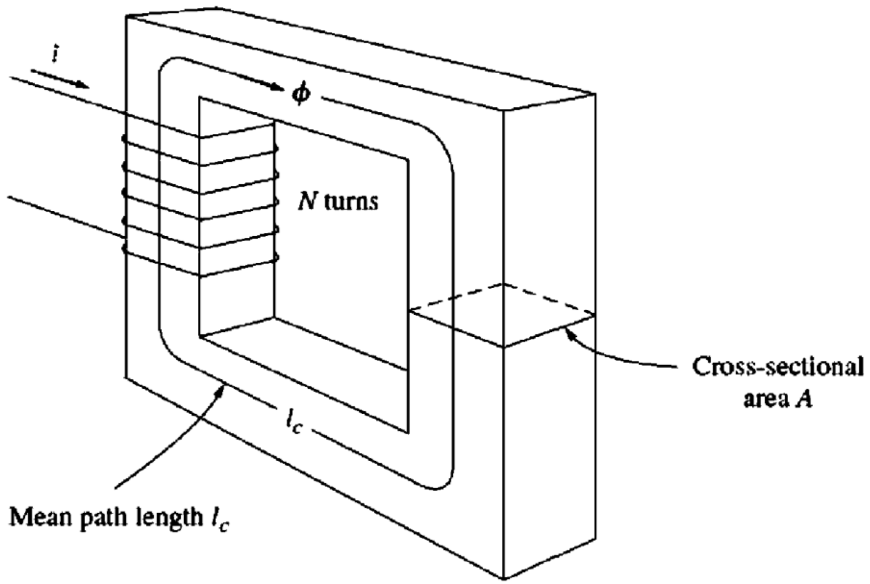
https://www.youtube.com/watch?v=TG__fe_FNf4&ab_channel=OpenMind

Simple Magnetic Circuit

- A rectangular core with a winding of N turns of wire wrapped about one leg of the core. If the core is composed of iron or certain other similar metals, essentially all the magnetic field produced by the current will remain inside the core, so the path of integration in Ampere's law is the mean path length of the core *lc*.

Magnetomotive force (*mmf*) is a quantity appearing in the equation for the magnetic flux in a magnetic circuit.

$$mmf:\ \mathcal{F} = Hl_c = Ni$$

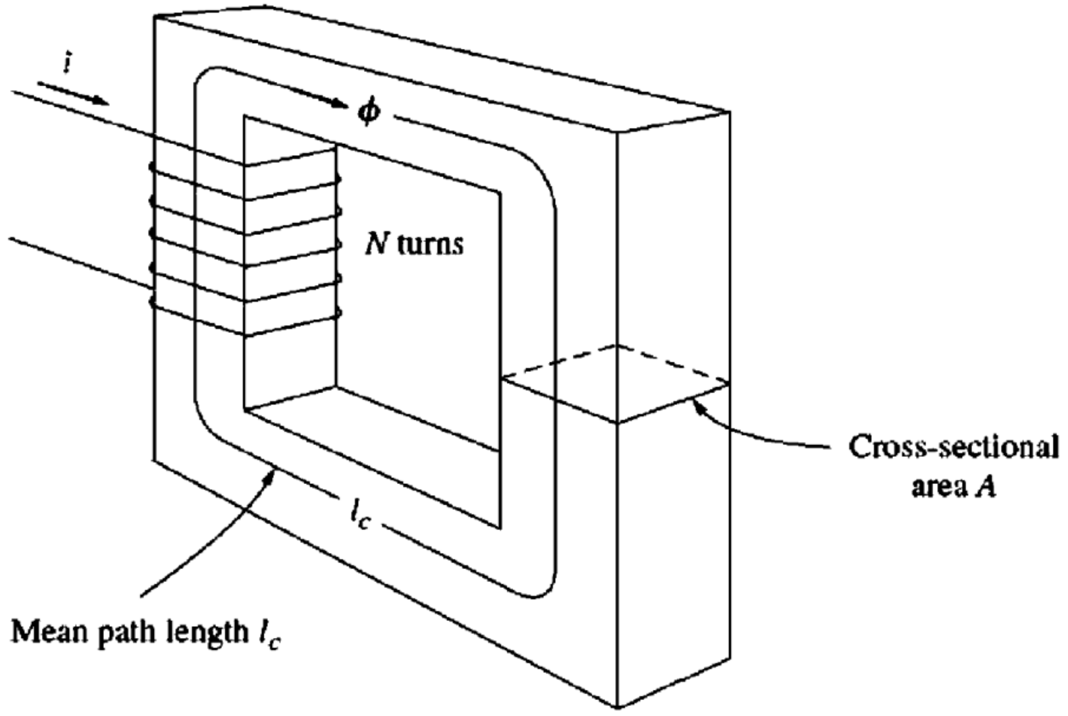


Simple Magnetic Circuit

- The current passing within the path of integration *i* is then *Ni*, since the coil of wire cuts the path of integration *N* times while carrying current *i*. Ampere's law thus becomes

$$Hl_c = Ni$$

$$H = \frac{Ni}{l_c}$$



Magnetic Flux Density and Permeability

- The magnetic field intensity H is in a sense a measure of the effort that a current is putting into the establishment of a magnetic field. The strength of the magnetic field flux produced in a core also depends on the material of the core.
- The relationship between the magnetic field intensity H and the resulting magnetic flux density B produced within a material is given by

$$B = \mu H$$

where

H = magnetic field intensity

μ = magnetic permeability of material

B = *resulting magnetic flux density produced*

Magnetic Field Intensity	*H* [Amp-turn/m]
Magnetic Flux	*Φ* [Wb] (Webers)
Magnetic Flux Density	*B* [Wb/m^2] = T (Tesla)

Relative Permeability - Magnetic Core Characteristics

- The units of magnetic field intensity are ampere-turns per meter, the units of permeability are henrys per meter, and the units of the resulting flux density are webers per square meter, known as teslas (T)
- The permeability of free space is called μ_0, and its value is

$$\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$$

- The permeability of any other material compared to the permeability of free space is called its relative permeability

$$\mu_r = \frac{\mu}{\mu_0}$$

Relative Permeability - Magnetic Core Characteristics

- **Permeability** is the measure of magnetization that a material obtains in response to an applied magnetic field

	Electrical Conductivity $\times 10^6$ *S/m*	*Relative Magnetic Permeability @ B=20 Gauss*
Cold rolled steel	10.0	100
Iron	10.0	200
Purified iron	10.0	5,000
4% Silicon iron	1.7	500
45 Permalloy	2.2	2,500
78 Permalloy	6.3	8,000
4-79 Permalloy	1.8	20,000
2-81 Permalloy	0.0001	125
Supermalloy	1.7	100,000
Mu Metal	1.6	20,000
Hiperco	4.0	650
Hypernik	2.0	4,500
Monimax	1.3	2,000
Sinimax	1.1	3,000
Permendur	14.3	800
2V Permendur	3.8	800

Magnetic Materials

Material	Relative Permeability μ_r
Vacuum	1
Air	1.0000004
Water	0.999992
Copper	0.999994
Aluminum	1.00002
Silver	0.99998
Nickel	600
Iron	5000
Carbon Steel	100
Transformer Steel	2000
Mumetal	50,000
Supermalloy	1,000,000

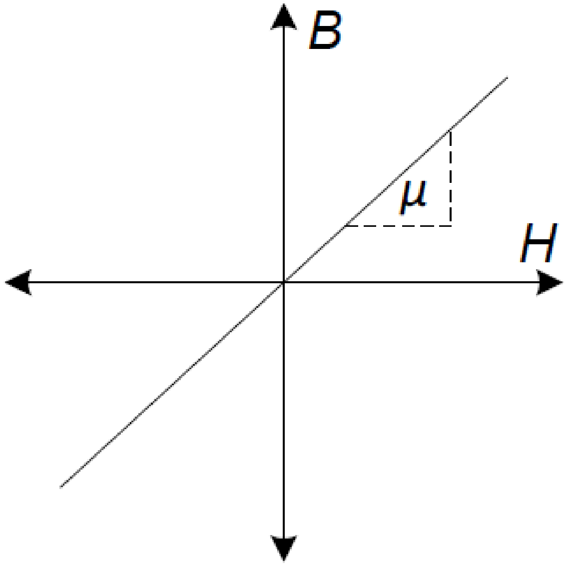
Note: Values can often vary depending on purity and processing.

http://en.wikipedia.org/wiki/Permeability_(electromagnetism)

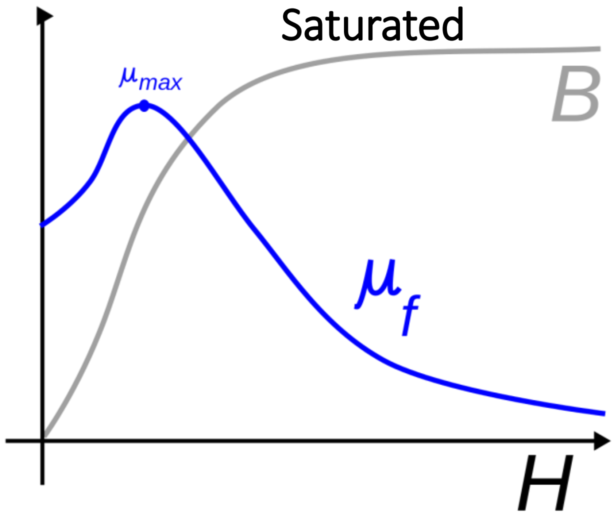
Relative Permeability - Magnetic Core Characteristics

- Relative permeability is a convenient way to compare the magnetizability of materials. For example, the steels used in modern machines have relative permeabilities of 2000 to 6000 or even more. This means that, for a given amount of current, 2000 to 6000 times more flux is established in a piece of steel than in a corresponding area of air.

$$B = \mu H = \frac{\mu N i}{l_c}$$



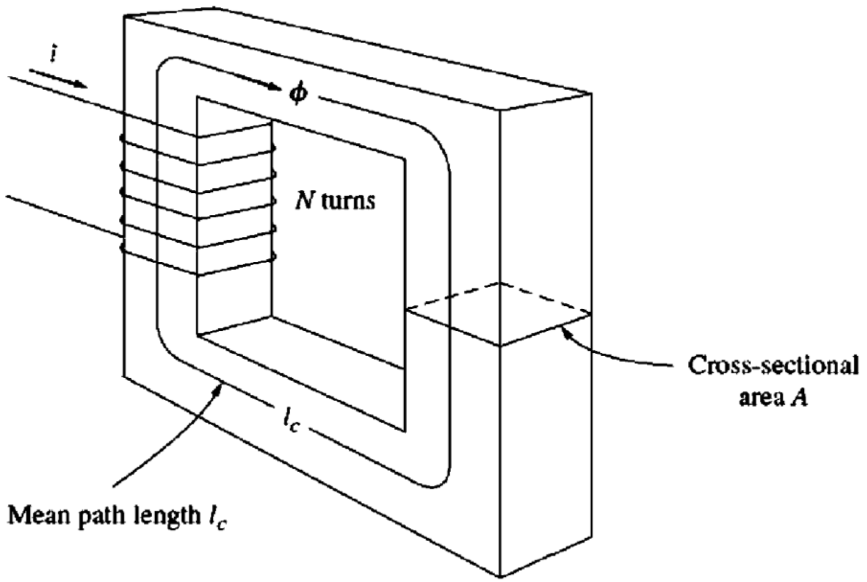
Ideal situation



Actual situation

Inside a Ferromagnetic Material

- Because the permeability of iron is so much higher than that of air, the great majority of the flux in an iron core remains inside the core instead of traveling through the surrounding air, which has much lower permeability.
- The small leakage flux that does leave the iron core is very important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.



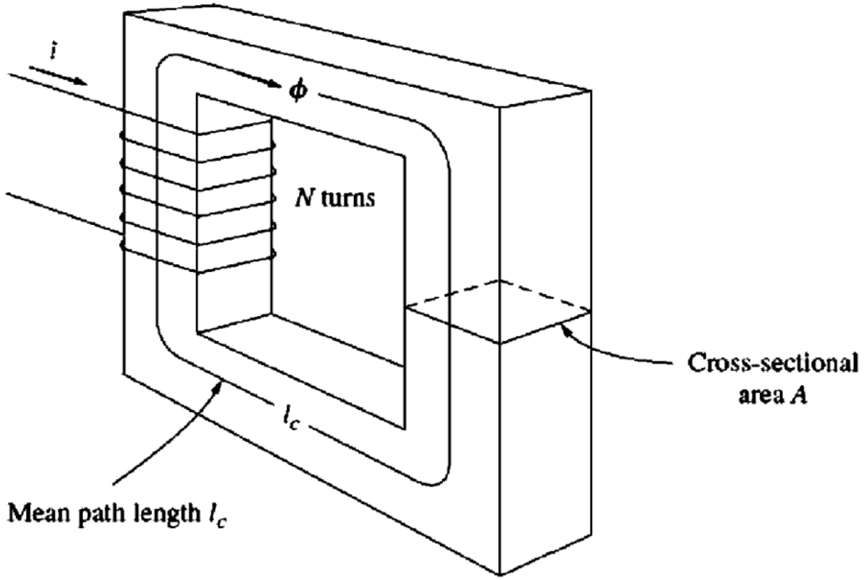
Inside a Ferromagnetic Material

- The total flux in a given area is given by

$$\phi = \int_A B \cdot dA$$

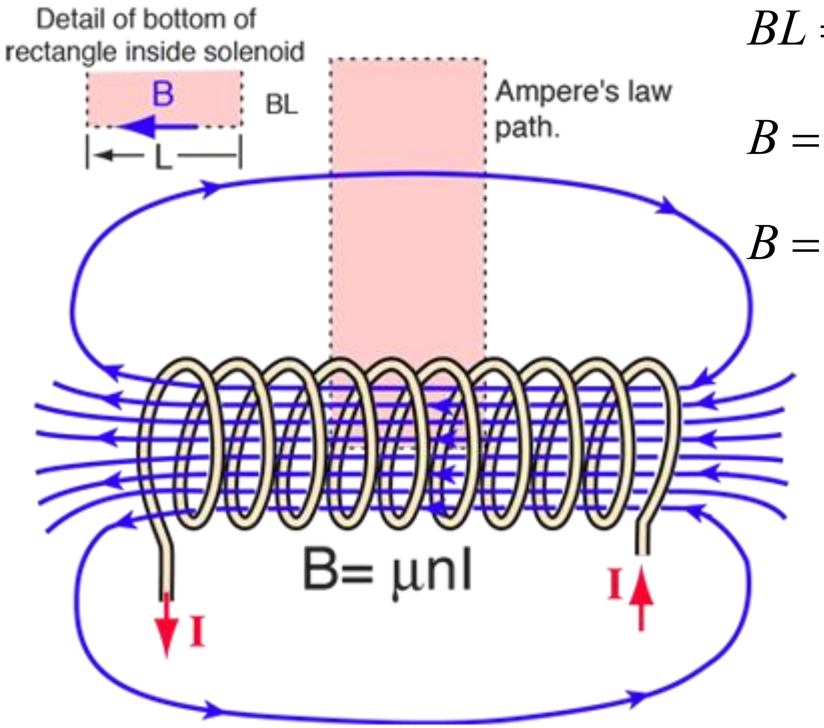
- where *dA* is the differential unit of area. If the flux density vector is perpendicular to a plane of area *A*, and if the flux density is constant throughout the area, then this equation reduces to

$$\phi = BA$$
$$= \frac{\mu N i A}{l_c}$$



Example 1: Magnetic Flux Density Calculation

- Solenoid Field from Ampere's Law



$$BL = \mu NI$$

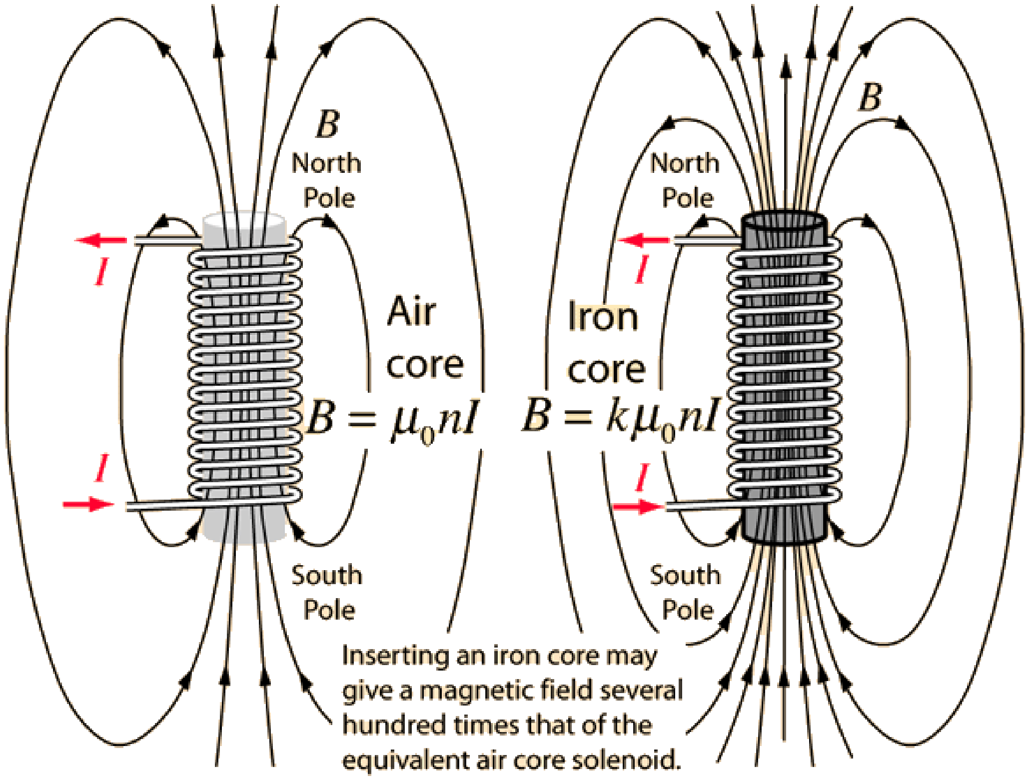
$$B = \mu \frac{N}{L} I$$

$$B = \mu nI$$

Without Iron inserted

$$B = k\mu_0 nI$$

k is the relative permeability of the iron, shows the magnifying effect of the iron core.

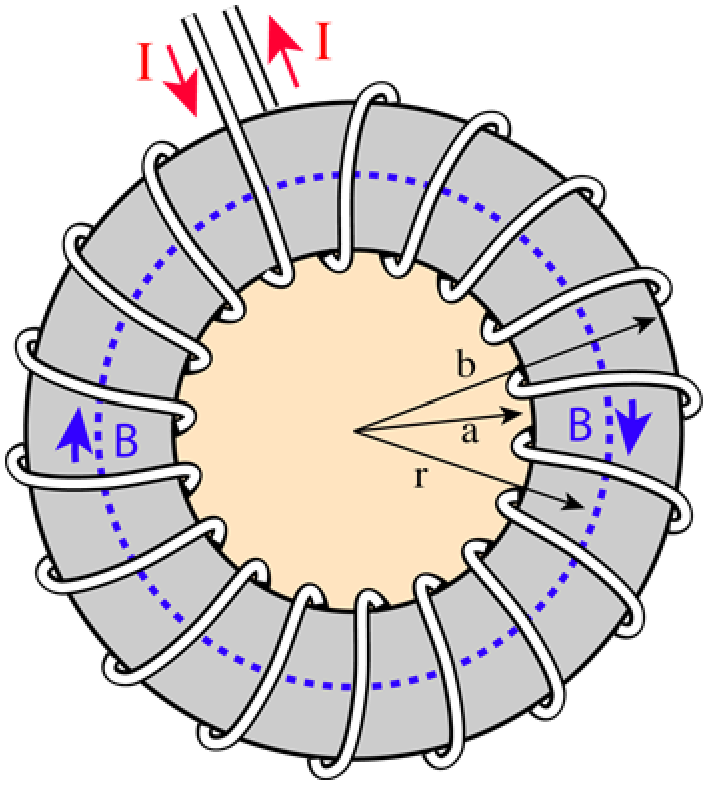


With Iron inserted

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/solenoid.html#c2

Example 2: Magnetic Flux Density Calculation

- Magnetic Field of Toroid



With Iron inserted

The current enclosed by the dashed line is just the number of loops times the current in each loop. Amperes law then gives the magnetic field by

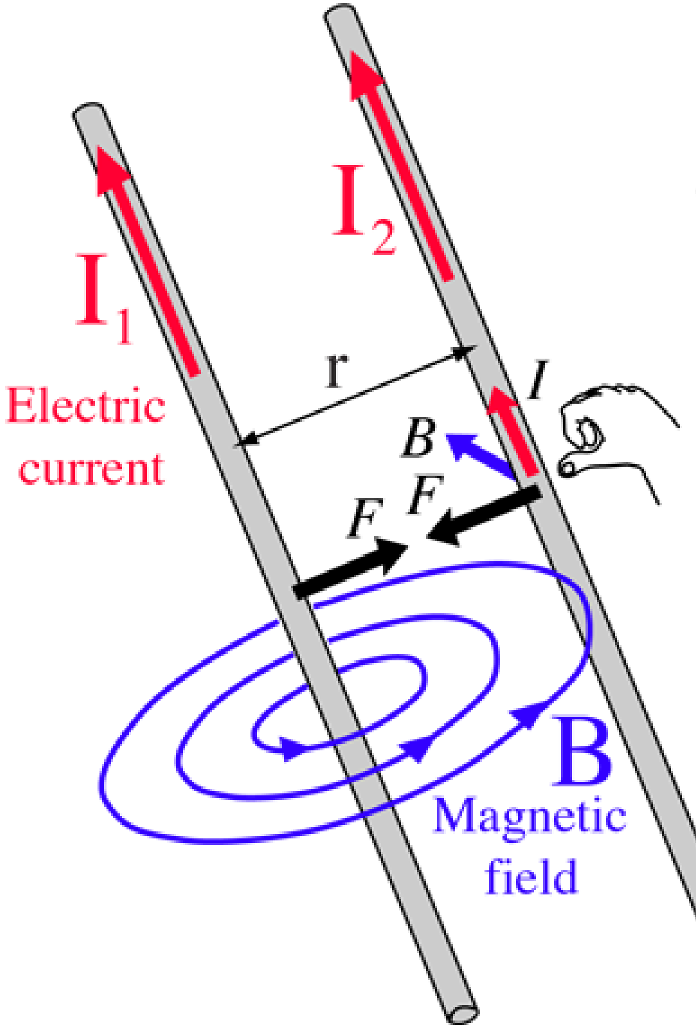
$$B 2\pi r = \mu N I$$

$$B = \frac{\mu N I}{2\pi r}$$

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/toroid.html#c1

Example 3: Magnetic Flux Density Calculation

- Magnetic Force Between Wires



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

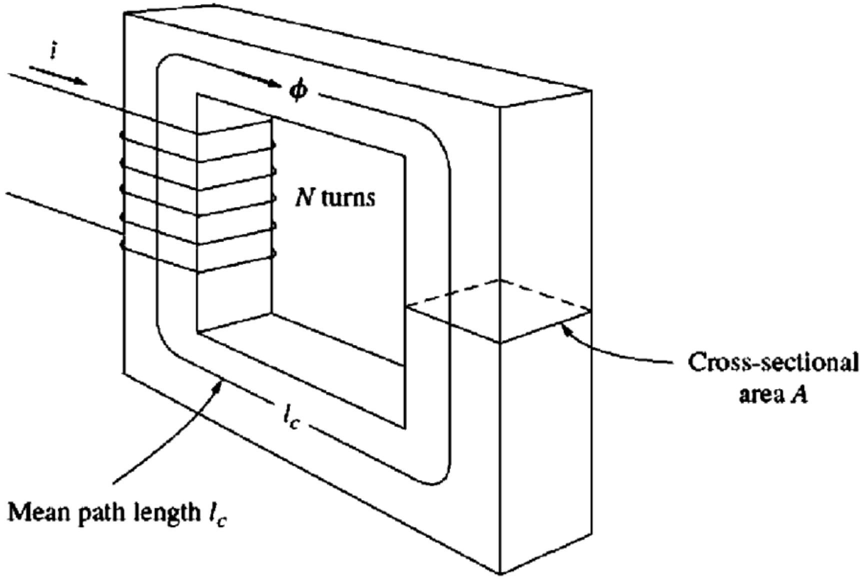
$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Source: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/wirfor.html#c1

Magnetic Circuit - Reluctance of Magnetic Bar

We've already known

$$\phi = BA = \frac{\mu N i A}{l_c} \quad \rightarrow \quad \phi = BA = \frac{\mu \mathcal{F} A}{l_c}$$

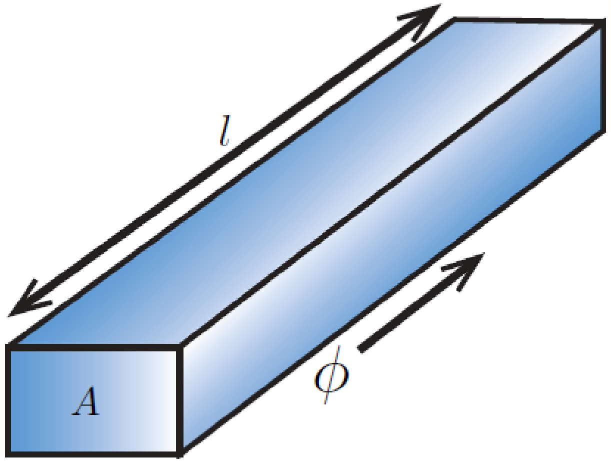


- Magnetic "OHM's LAW"

$$mmf: \ \mathcal{F} = Ni = \Phi \Re$$

$$\boxed{\Re = \frac{l}{\mu A}}$$

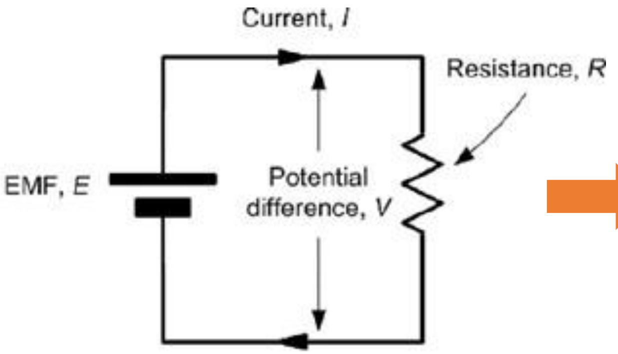
  - High material permeability can result in low core reluctance.

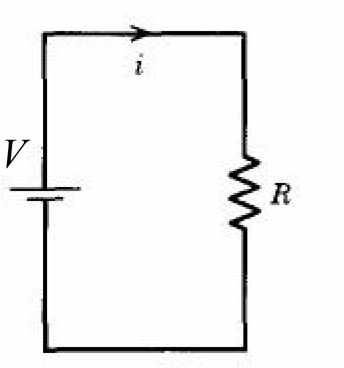


The reluctance $\mathcal{R}$ of a magnetic path depends on the mean length l, the area A, and the permeability μ of the material.

Magnetic Circuit - Electrical Circuit Analogy

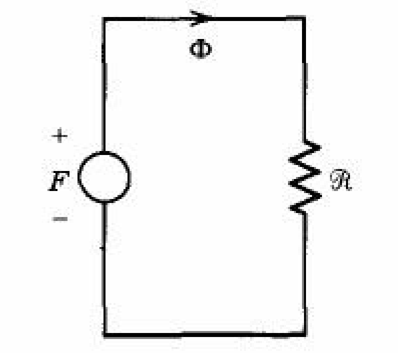
- Electric circuit

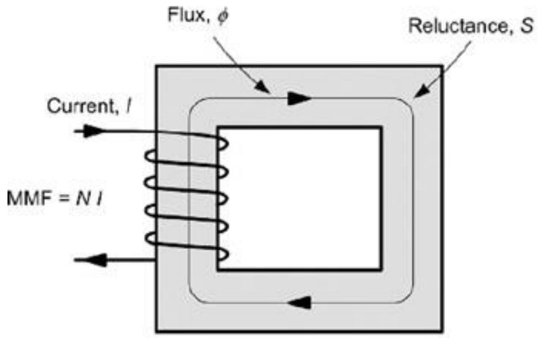




$$i = \frac{V}{R}$$

- Magnetic circuit



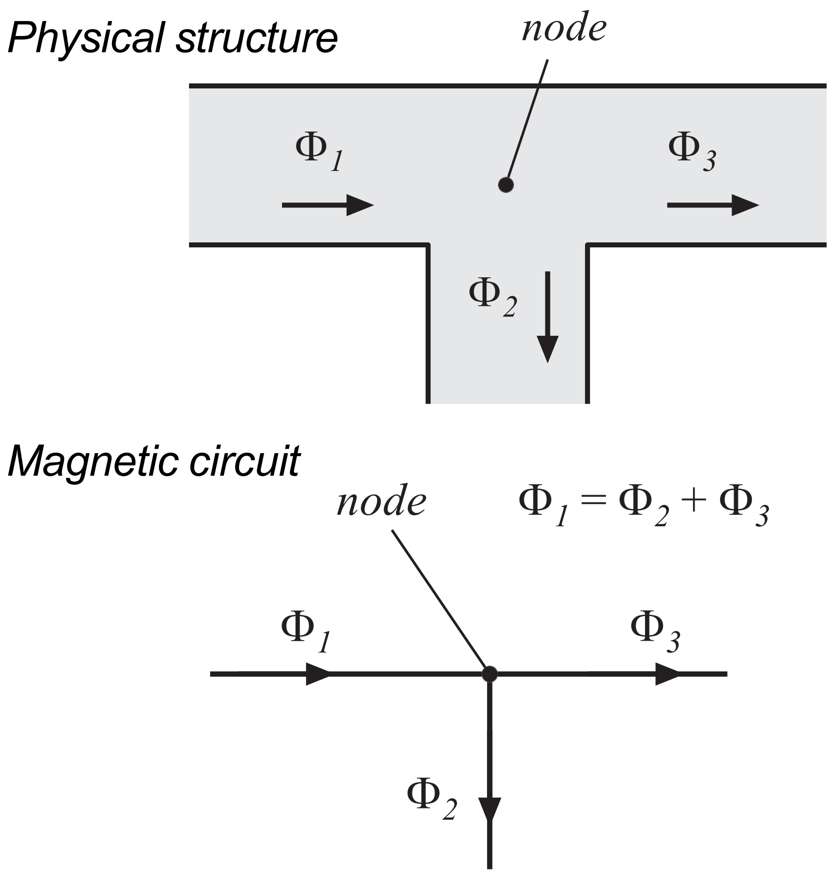


$$\phi = \frac{\mathcal{F}}{\mathcal{R}}$$

Electrical	Magnetic
Voltage v	Magnetomotive Force $\Im = Ni$
Current i	Magnetic Flux ϕ
Resistance R	Reluctance $\Re$
Conductivity $1/\rho$	Permeability μ
Current Density J	Magnetic Flux Density B
Electric Field E	Magnetic Field Intensity H

Review of basic magnetics

Magnetic analog of Kirchoff's current law



Divergence of ***B*** = 0

Flux lines are continuous and cannot end

The total flux entering a node must be zero

This relationship is equivalent to Kirchoff's current law.

Review of basic magnetics

Magnetic analog of Kirchoff's voltage law

Follows from Ampere's law:

$$\oint \boldsymbol{H} \cdot \boldsymbol{dl} = \textit{total current passing through interior of path}$$

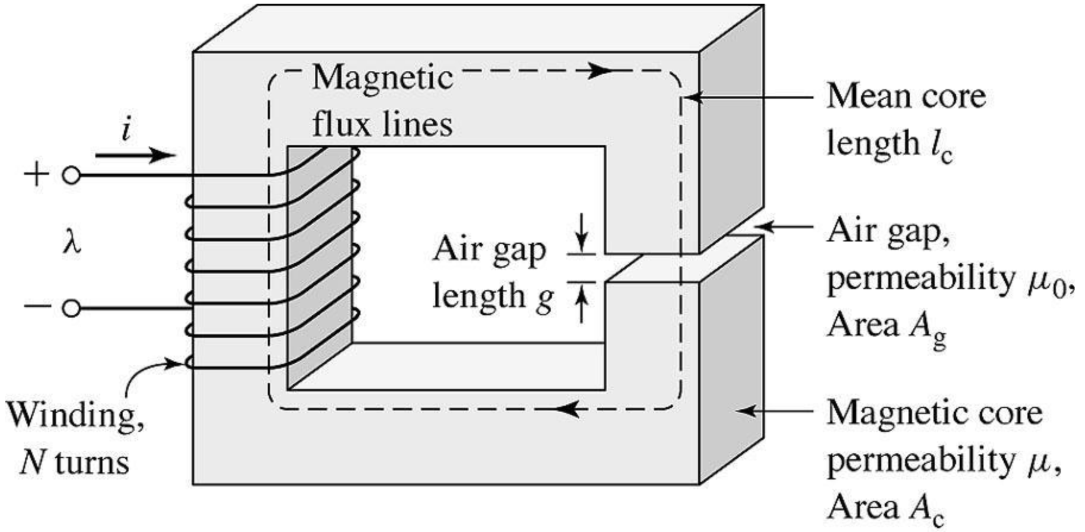
Left-hand side: sum of MMF's across the reluctances around the closed path

Right-hand side: currents in windings are sources of MMF's. An *n*-turn winding carrying current *i(t)* is modeled as an MMF (voltage) source, of value *ni(t)*.

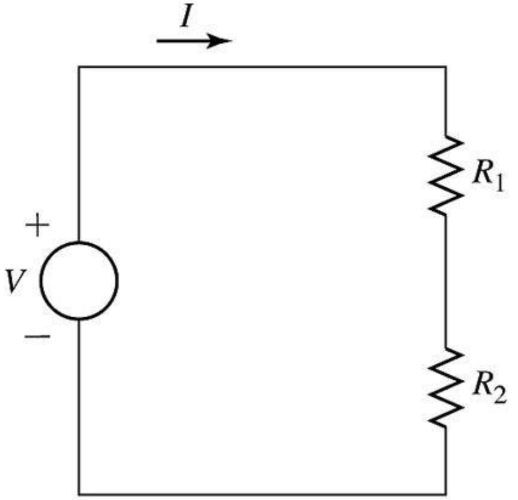
Total MMF's around the closed path add up to zero.

Magnetic Circuit with Air Gap

Provided the air-gap length g is sufficiently small, the configuration can be analyzed as a magnetic circuit with two series components both carrying the same flux.

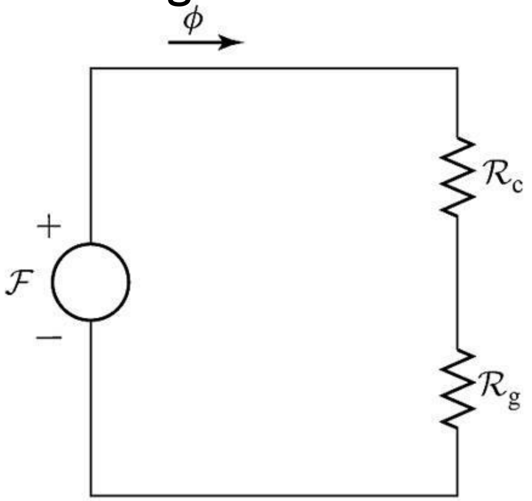


Electric circuit



$$I = \frac{V}{(R_1 + R_2)}$$

Magnetic circuit



$$\phi = \frac{\mathcal{F}}{(\mathcal{R}_c + \mathcal{R}_g)}$$

$$\mathcal{R}_c = \frac{l_c}{\mu A_c}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g}$$

Reluctance

$$\phi = \frac{\mathcal{F}}{\mathcal{R}} \rightarrow \phi = \frac{\mathcal{F}}{\mathcal{R}_c + \mathcal{R}_g} \rightarrow \phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}} \rightarrow \boxed{B_c = \frac{\phi}{A_c}}$$

Flux

Flux density in Core

Magnetic Circuit with Air Gap

$$\phi = \frac{\mathcal{F}}{\frac{l_c}{\mu A_c} + \frac{g}{\mu_0 A_g}}$$

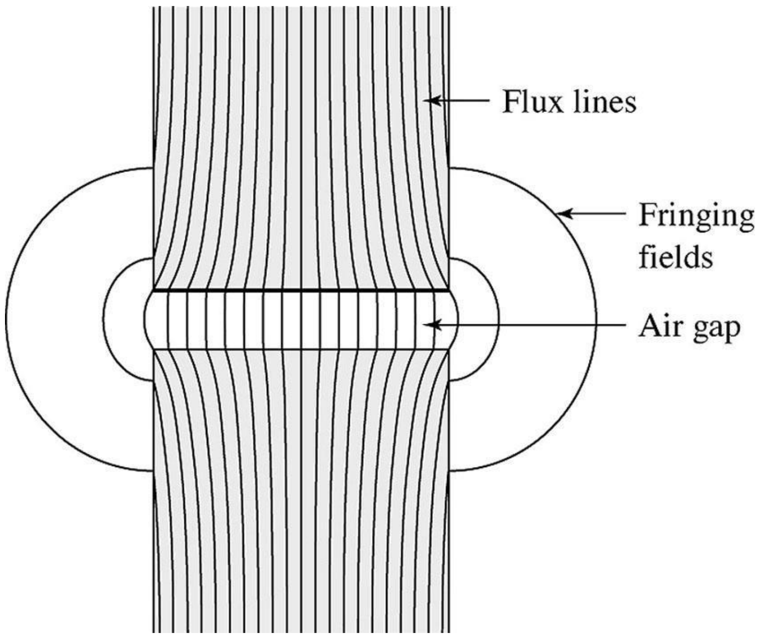
We see that high material permeability can result in low core reluctance, which can often be made much smaller than that of the air gap, thus

$$(\mu A_c/l_c) \gg (\mu_0 A_g/g),\ \mathcal{R}_c \ll \mathcal{R}_g$$

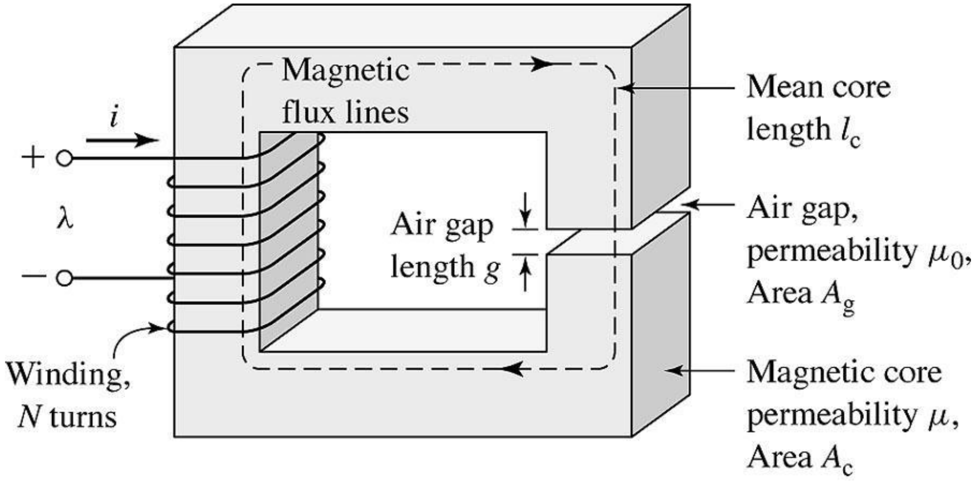
In this case, the reluctance of the core can be neglected and the flux can be found

$$\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F}\mu_0 A_g}{g} = Ni\,\frac{\mu_0 A_g}{g}$$

In practical systems, the magnetic field lines "fringe" outward somewhat as they cross the air gap. Provided this fringing effect is not excessive, the magnetic-circuit concept remains applicable. The effect of these fringing fields is to increase the effective cross-sectional area *Ag* of the air gap. In this course, the effect of fringing fields is ignored, thus *Ag=Ac*.



Air-gap fringing fields.



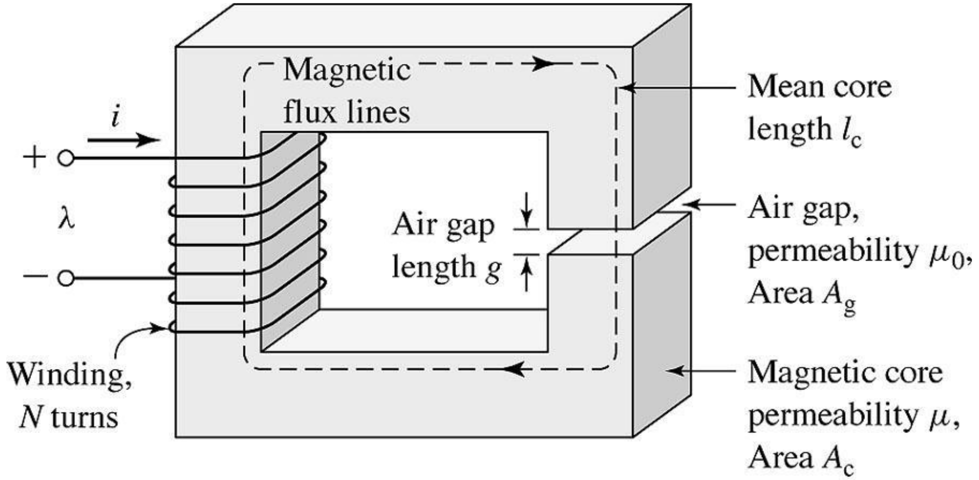
Example 4: Magnetic Circuit with Air Gap

The magnetic circuit shown in right figure has dimensions $A_c = A_g = 9$ cm^2, $g = 0.050$ cm, $l_c = 30$ cm, and $N = 500$ turns. Assume the value $\mu_r = 70{,}000$ for core material.

(a) Find the reluctances $\Re_c$ and $\Re_g$.

(b) For the condition that the magnetic circuit is operating with $B_c = 1.0$ T, find the flux ϕ

(c) Find the current i based on (b).



❑ Solution

(a) $$\mathcal{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} = \frac{0.3}{70,000\,(4\pi \times 10^{-7})(9 \times 10^{-4})} = 3.79 \times 10^3 \ \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

$$\mathcal{R}_g = \frac{g}{\mu_0 A_g} = \frac{5 \times 10^{-4}}{(4\pi \times 10^{-7})(9 \times 10^{-4})} = 4.42 \times 10^5 \ \frac{\text{A} \cdot \text{turns}}{\text{Wb}}$$

Be careful with the Units

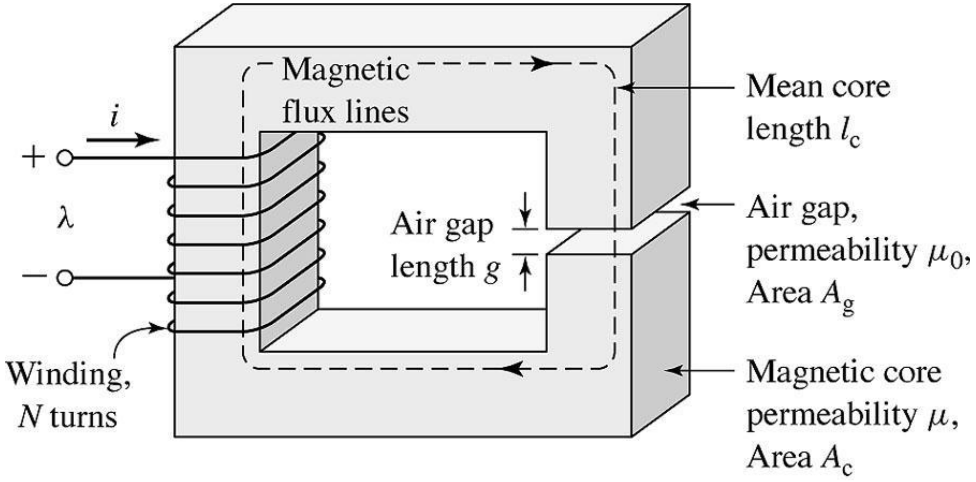
(b) $$\phi = B_c A_c = 1.0(9 \times 10^{-4}) = 9 \times 10^{-4} \text{ Wb}$$

(c) $$i = \frac{\mathcal{F}}{N} = \frac{\phi(\mathcal{R}_c + \mathcal{R}_g)}{N} = \frac{9 \times 10^{-4}(4.46 \times 10^5)}{500} = 0.80 \text{ A}$$

Flux Linkage and Inductance

Based on Faraday's Law of Induction, the time-varying flux will induce a voltage (*emf*) on the coil.

$$emf = -N\frac{d\Phi_{mag}}{dt}$$



Change the pattern and then use induced voltage *e* to represent the *emf*.

$$e = N\frac{d\varphi}{dt} = \frac{d\lambda}{dt}$$

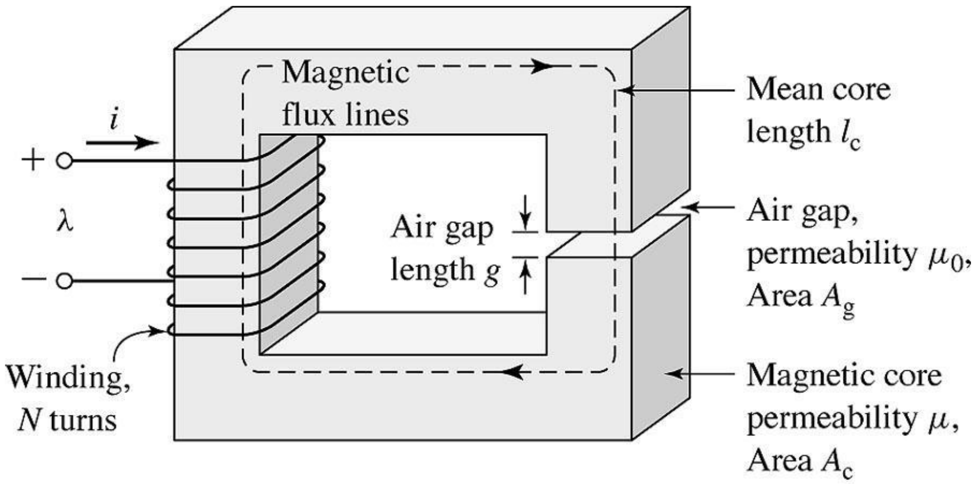
where λ is the ***flux linkage*** of the winding and is defined as $\boxed{\lambda = N\varphi}$

Flux linkage is measured in units of webers (or equivalently weber-turns). Note that we have chosen the symbol φ to indicate the instantaneous value of a time-varying flux ϕ.

Flux Linkage and Inductance

For a magnetic circuit composed of magnetic material of constant magnetic permeability or which includes a dominating air gap, the relationship between λ and i will be linear and we can define the ***inductance L*** as

$$\boxed{L = \frac{\lambda}{i}}$$



For the magnetic circuit with an air gap, we have

$$\boxed{\phi \approx \frac{\mathcal{F}}{\mathcal{R}_g} = \frac{\mathcal{F}\mu_0 A_g}{g} = Ni\,\frac{\mu_0 A_g}{g}} \qquad \boxed{\lambda = N\varphi}$$

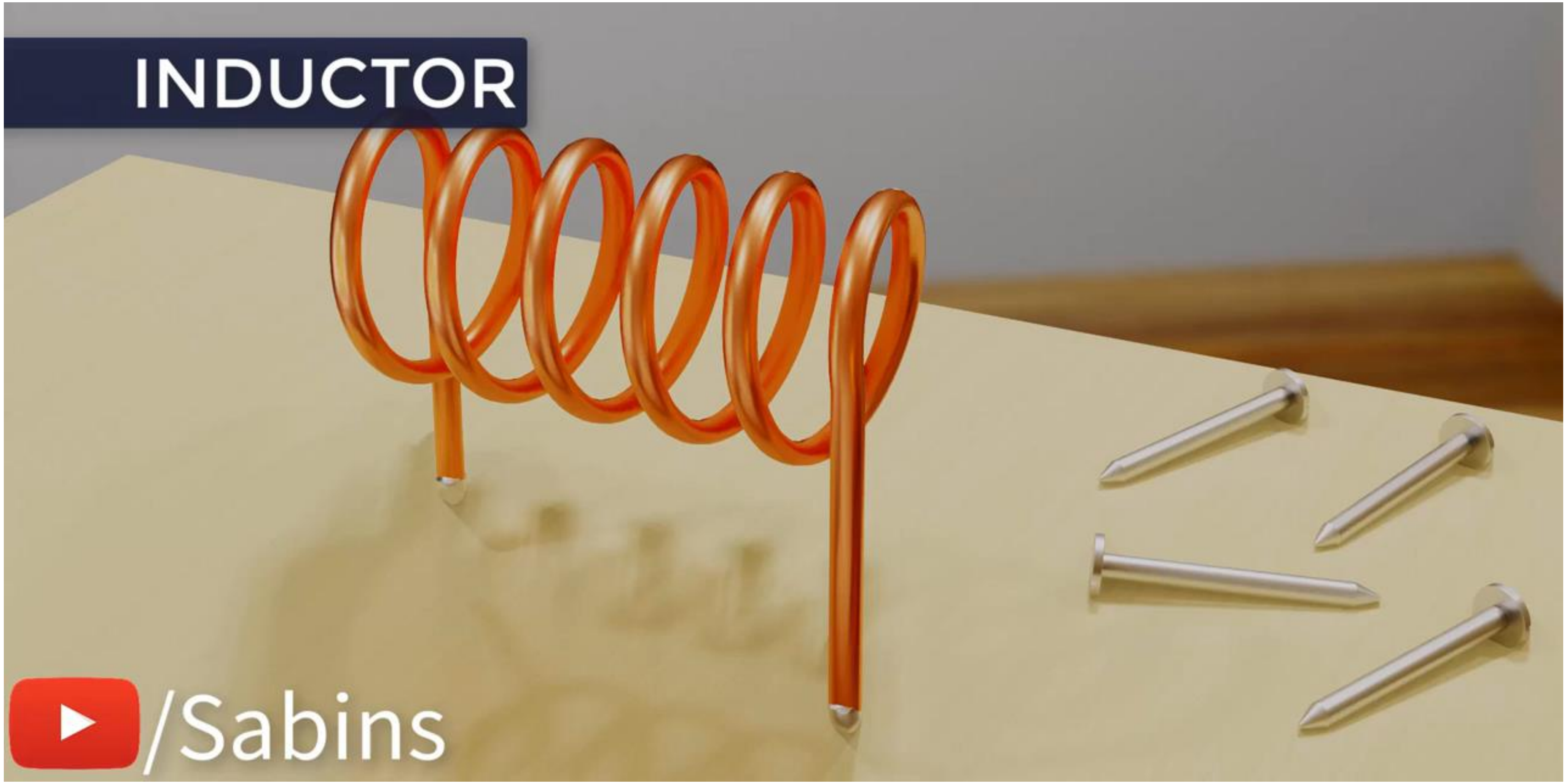
Under the assumption that the reluctance of the core is negligible as compared to that of the air gap, the inductance of the winding

$$\boxed{L = \frac{N^2}{(g/\mu_0 A_g)} = \frac{N^2 \mu_0 A_g}{g}}$$

Inductance is proportional to the square of the number of turns, to a magnetic permeability and to a cross-sectional area and is inversely proportional to a length. L is irrelevant to current i even defined by i.

Inductance is measured in *henrys* (H) or *weber-turns per ampere.*

What is Inductor



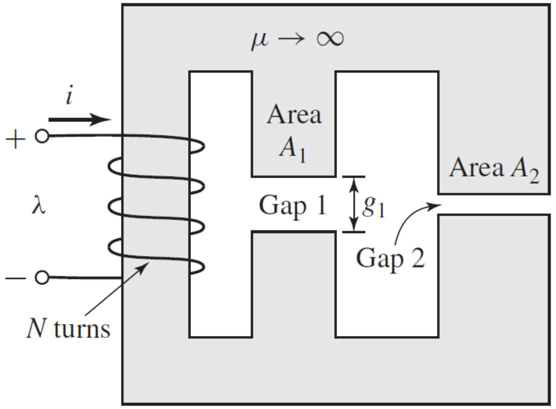
https://www.youtube.com/watch?v=d73e3QiMdSU&ab_channel=Sabins

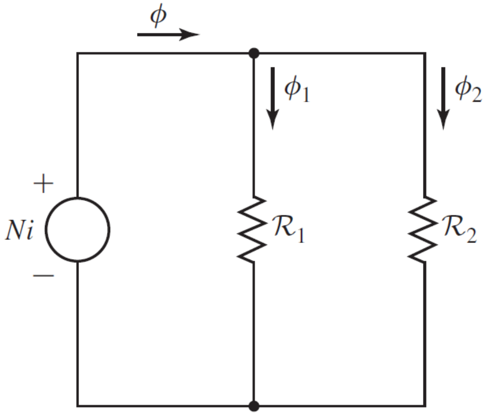
Example 5: Inductance Calculation with Air Gaps

The magnetic circuit of right figure consists of an N-turn winding on a magnetic core of infinite permeability with two parallel air gaps of lengths g_1 and g_2 and areas A_1 and A_2, respectively.

(a) Find the inductance of the winding

(b) Find the flux density B_1 in gap 1 when the winding is carrying a current i. Neglect fringing effects at the air gap.





❑ Solution

(a) The permeability of the magnetic core is infinite, then the core reluctance can be neglected. Thus, the total reluctance is equal to the parallel combination of the two gap reluctances.

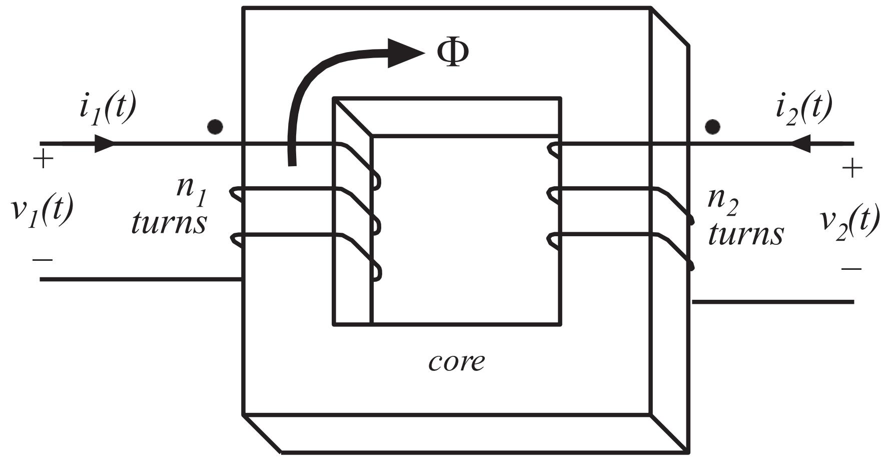
$$\mathcal{R}_1 = \frac{g_1}{\mu_0 A_1} \qquad \mathcal{R}_2 = \frac{g_2}{\mu_0 A_2} \qquad \phi = \frac{Ni}{\frac{\mathcal{R}_1 \mathcal{R}_2}{\mathcal{R}_1 + \mathcal{R}_2}} \Rightarrow L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 \mathcal{R}_2}$$

$$= \mu_0 N^2 \left(\frac{A_1}{g_1} + \frac{A_2}{g_2} \right)$$

(b) From the equivalent circuit, one can see that

$$\phi_1 = \frac{Ni}{\mathcal{R}_1} = \frac{\mu_0 A_1 Ni}{g_1} \Rightarrow B_1 = \frac{\phi_1}{A_1} = \frac{\mu_0 Ni}{g_1}$$

Transformer modeling



Two windings, no air gap:

The reluctance:

$$R = \frac{l_m}{\mu A_c}$$

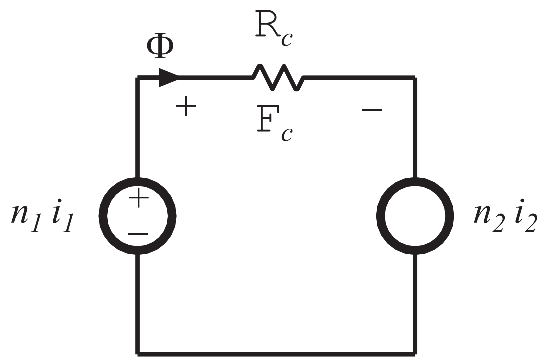
The MMF:

$$F_c = n_1 i_1 + n_2 i_2$$

The magnetic circuit:

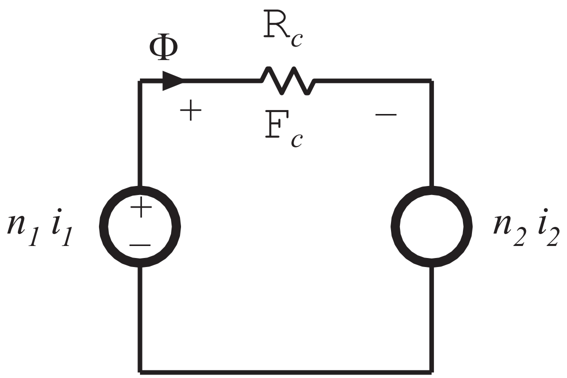
$$\Phi_1 R_1 = n_1 i_1 + n_2 i_2$$

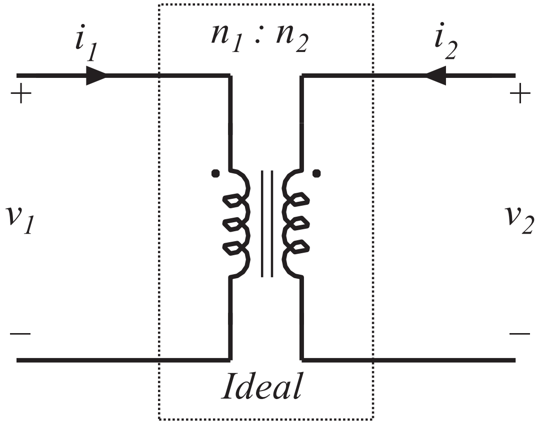
Therefore, based on the magnetic circuit equation.
The magnetic circuit model can be represented as:



Transformer modeling

The ideal transformer





In the ideal transformer, the core reluctance R approaches zero.

MMF F_c = ΦR also approaches zero. We then obtain

$$0 = n_1 i_1 + n_2 i_2$$

Also, by Faraday's law,

$$v_1(t) = n_1 \frac{d\Phi(t)}{dt} \qquad v_2(t) = n_2 \frac{d\Phi(t)}{dt}$$

Eliminate Φ :

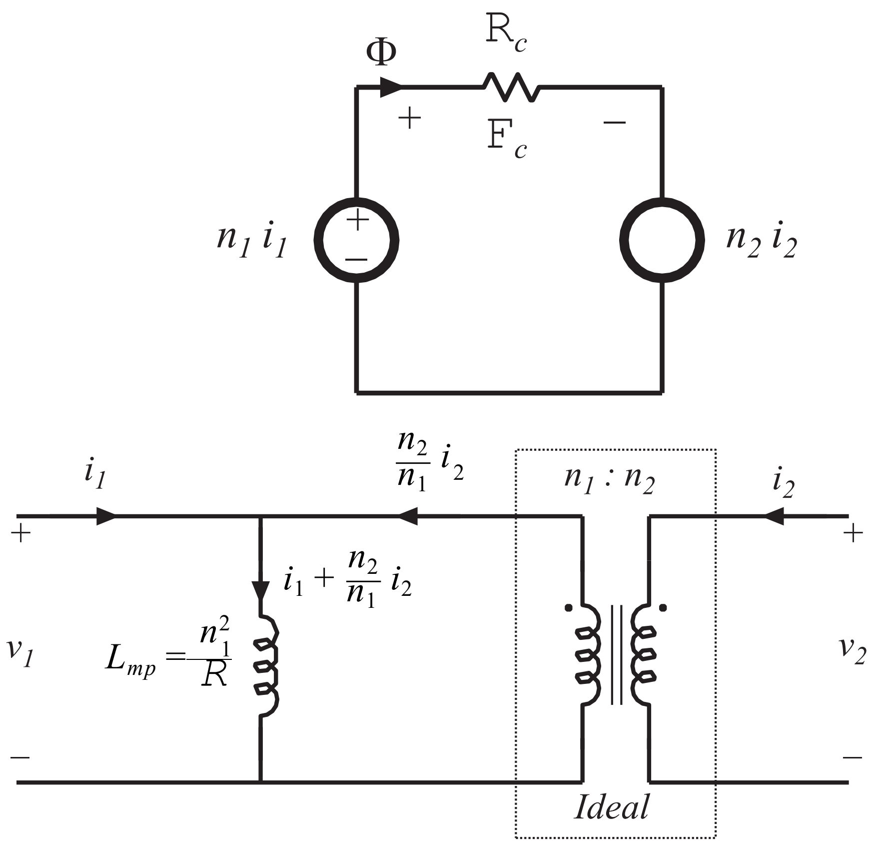
$$\frac{d\Phi}{dt} = \frac{v_1}{n_1} = \frac{v_2}{n_2}$$

Ideal transformer equations:

$$\frac{v_1}{n_1} = \frac{v_2}{n_2} \quad \text{and} \quad 0 = n_1 i_1 + n_2 i_2$$

Transformer modeling

The magnetizing inductance



In reality, the permeability of a core material can not be infinite.

Therefore, for nonzero core reluctance, we obtain

$$\Phi \Re = n_1 i_1 + n_2 i_2 \qquad v_1 = n_1 \frac{d\Phi}{dt}$$

Eliminate Φ:

$$v_1 = \frac{n_1^2}{R} \frac{d}{dt}\left[i_1 + \frac{n_2}{n_1} i_2 \right]$$

This equation is of the form:

$$v_1 = L_{mp} \frac{di_{mp}}{dt}$$

The magnetizing inductance and current can be obtained:

$$L_{mp} = \frac{n_1^2}{R} \qquad i_{mp} = i_1 + \frac{n_2}{n_1} i_2$$

Transformer modeling

Magnetizing inductance: discussion

- Models magnetization of core material.
- A real, physical inductor, that exhibits saturation and hysteresis.
- If the secondary winding is disconnected:

  we are left with the primary winding on the core primary winding then behaves as

  an inductor.

  The resulting inductor is the magnetizing inductance, referred to the primary winding.
- Magnetizing current causes the ratio of winding currents to differ from the turns ratio.

Transformer modeling

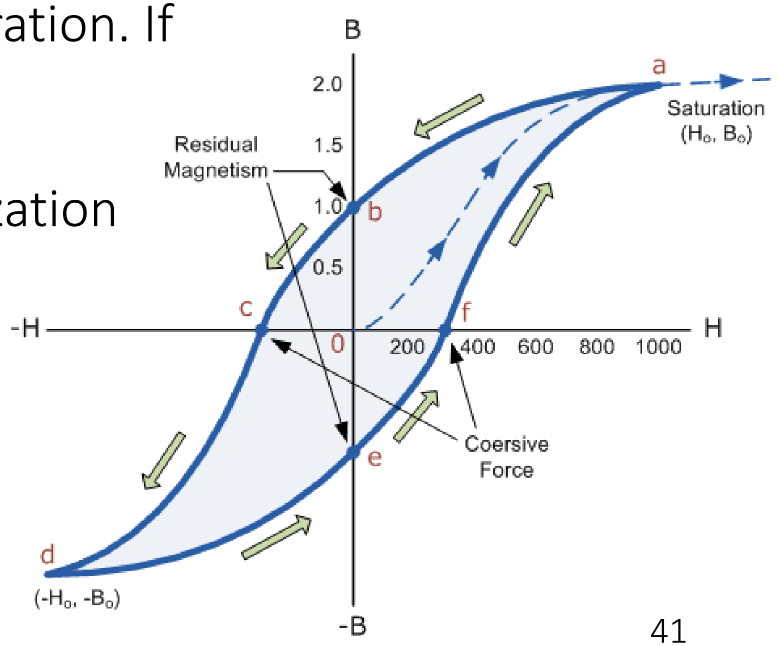
Transformer saturation

- Saturation occurs when core flux density *B(t)* exceeds saturation flux density B_{sat}.
- When the core saturates, the magnetizing current becomes large, the impedance of the magnetizing inductance becomes small, and the windings are effectively shorted out.
- Large winding currents $i_1(t)$ and $i_2(t)$ ***do not*** necessarily lead to saturation. If

$$0 = n_1 i_1 + n_2 i_2$$

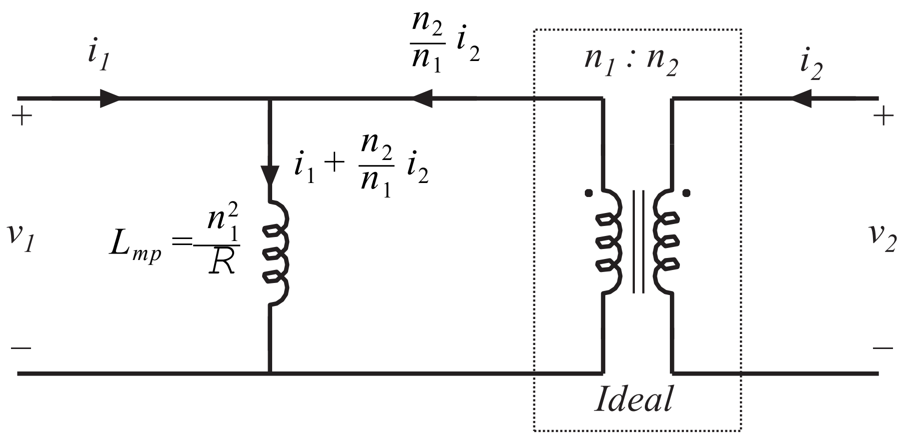
then the magnetizing current is zero, and there is no net magnetization of the core.

- Saturation is caused by excessive applied volt-seconds.



Transformer modeling

Saturation vs. applied volt-seconds



Magnetizing current depends on the integral of the applied winding voltage:

$$i_{mp}(t) = \frac{1}{L_{mp}} \int v_1(t)dt$$

Flux density is proportional: $L_{mp} = \dfrac{n_1^2}{R}$

$$B(t) = \frac{1}{n_1 A_c} \int v_1(t)dt$$

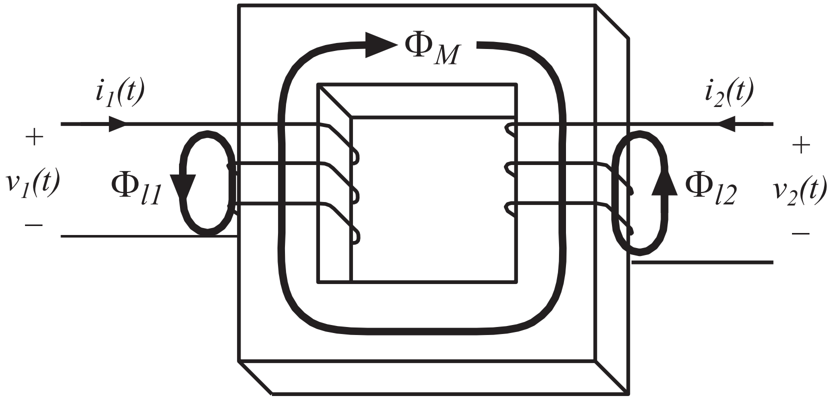
Flux density becomes large, and core saturates, when the applied volt-seconds λ_1 are too large, where

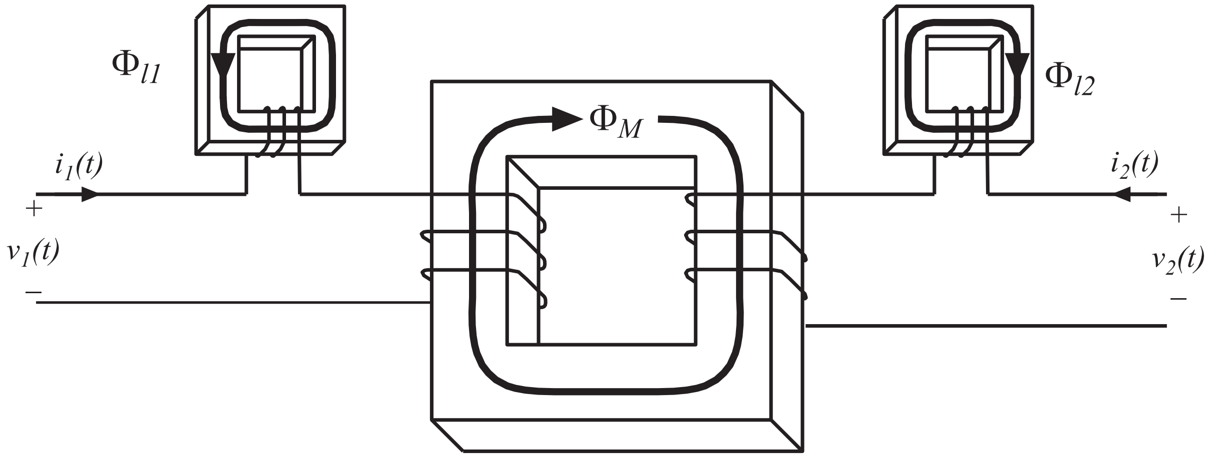
$$\lambda_1 = \int_{t_1}^{t_2} v_1(t)dt$$

limits of integration chosen to coincide with positive portion of applied voltage waveform.

Transformer modeling

Leakage inductances

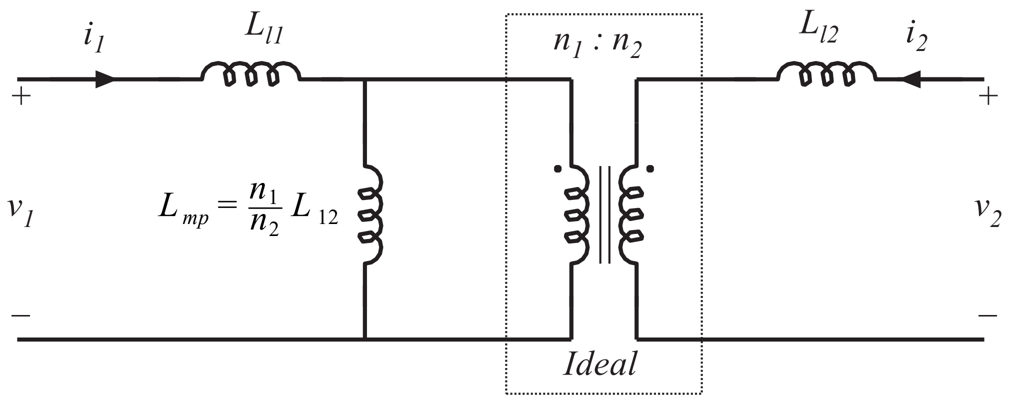




- In reality, magnetic flux is flowing in the vicinity of the windings and does not follow the core reluctance. This is called leakage flux.
- The leakage inductance, similar to the magnetization inductance, can be modeled with a physical inductor.
- The magnetic circuit can be represented with the leakage inductors and the transformer with the magnetization inductance.

Transformer modeling

Transformer model, including leakage inductance



$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} L_{11} L_{12} \\ L_{12} L_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix}$$

The mutual inductance:

$$L_{12} = \frac{n_1 n_2}{R} = \frac{n_2}{n_1} L_{mp}$$

primary and secondary self-inductances

$$L_{11} = L_{l1} + \frac{n_1}{n_2} L_{12}$$

$$L_{22} = L_{l2} + \frac{n_2}{n_1} L_{12}$$

effective turns ratio $n_e = \sqrt{\frac{L_{22}}{L_{11}}}$

coupling coefficient $k = \frac{L_{12}}{\sqrt{L_{11} L_{22}}}$

Loss mechanisms in magnetic devices

Low-frequency losses: DC copper loss

Core loss: hysteresis loss

High-frequency losses: the skin effect

Core loss: classical eddy current losses Eddy current losses in ferrite cores

High frequency copper loss: the proximity effect

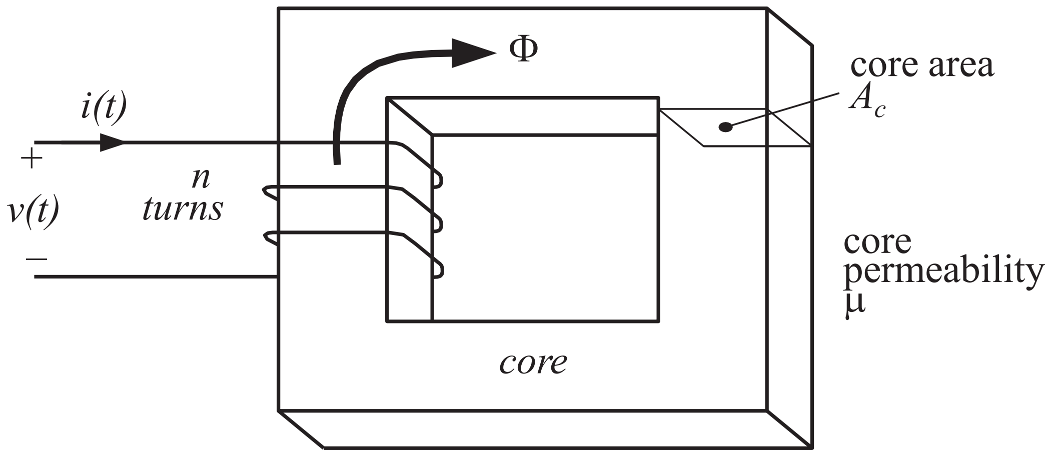
Proximity effect: high frequency limit

MMF diagrams, losses in a layer, and losses in basic multilayer windings

Effect of PWM waveform harmonics

Loss mechanisms in magnetic devices

Core loss



Energy per cycle *W* flowing into *n*- turn winding of an inductor, excited by periodic waveforms of frequency *f*:

$$W = \int_{\text{one cycle}} v(t)i(t)dt$$

Relate winding voltage and current to core *B* and *H* via Faraday's law and Ampere's law:

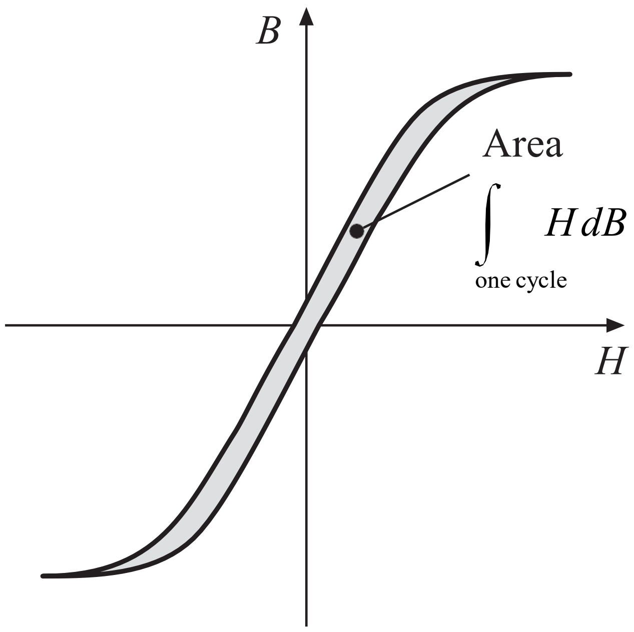
$$v(t) = nA_c \frac{dB(t)}{dt} \qquad H(t)l_m = ni(t)$$

Substitute into integral:

$$\begin{aligned} \mathrm{W} &= \int_{\text{one cycle}} \left(nA_c \frac{dB(t)}{dt} \right) \left(\frac{H(t)l_m}{n} \right) dt \\ &= (A_c l_m) \int_{\text{one cycle}} H\,dB \end{aligned}$$

Loss mechanisms in magnetic devices

Core loss: Hysteresis loss



The integrated energy:

$$W = (A_c l_m) \int_{\text{one cycle}} H dB$$

The term $A_c l_m$ is the volume of the core, while the integral is the area of the *B*-*H* loop.

(energy lost per cycle) = (core volume) (area of *B*-*H* loop)

$$P_H = (f)(A_c l_m) \int_{\text{onecycle}} H dB$$

Hysteresis loss is directly proportional to applied frequency.

Loss mechanisms in magnetic devices

Modeling hysteresis loss

- Hysteresis loss varies directly with applied frequency
- Dependence on maximum flux density: how does the area of *B-H* loop depend on maximum flux density (and on applied waveforms)? Emperical equation (Steinmetz equation):

$$P_H = K_H f B^{\alpha}{}_{\max} (core volume)$$

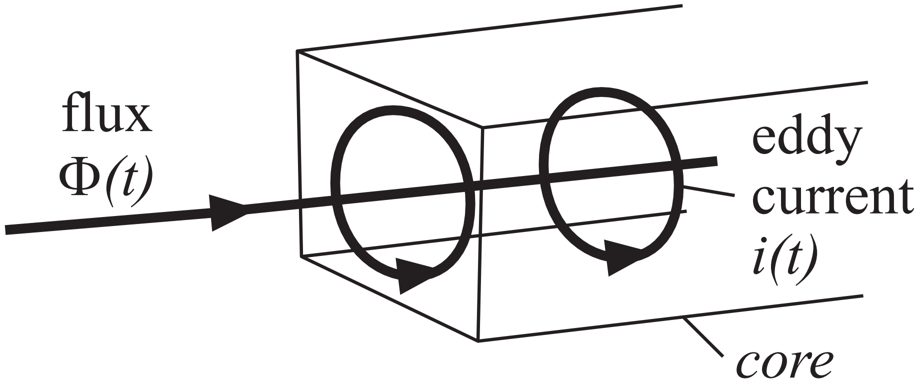
The parameters K_H and α are determined experimentally.

The dependence of P_H on B_{max} is predicted by the theory of magnetic domains.

Loss mechanisms in magnetic devices

Core loss: eddy current loss

Magnetic core materials are reasonably good conductors of electric current. Hence, according to Lenz's law, magnetic fields within the core induce currents ("eddy currents") to flow within the core. The eddy currents flow such that they tend to generate a flux that opposes changes in the core flux $\Phi(t)$. The eddy currents tend to prevent flux from penetrating the core.



Eddy current loss $i^2(t)R$

Loss mechanisms in magnetic devices

Modeling eddy current loss

- Ac flux $\Phi(t)$ induces voltage $v(t)$ in core, according to Faraday's law. Induced voltage is proportional to the derivative of $\Phi(t)$. In consequence, the magnitude of induced voltage is directly proportional to excitation frequency f.
- If core material impedance Z is purely resistive and independent of frequency, $Z = R$, then eddy current magnitude is proportional to the voltage: $i(t) = v(t)/R$. Hence the magnitude of $i(t)$ is directly proportional to excitation frequency f.
- Eddy current power loss $i^2(t)R$ then varies with the square of excitation frequency f.
- Classical Steinmetz equation for eddy current loss:

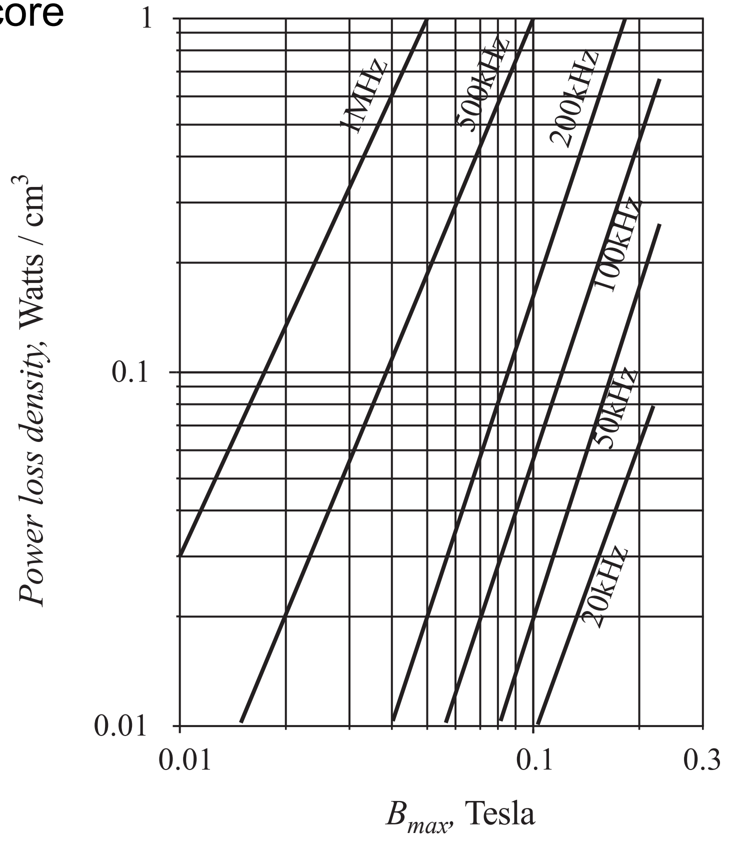
$$P_E = K_E f^2 B_{max}^2$$

- Ferrite core material impedance is capacitive. This causes eddy current power loss to increase as f^4.

Loss mechanisms in magnetic devices

Total core loss: manufacturer's data

Ferrite core material



Empirical equation, at a fixed frequency:

$$P_{fe} = K_{fe} B_{max}^{\beta} A_c l_m$$

- The core loss typically increases exponentially with the increase of flux density and efficiency

- At very high frequencies, like 1 MHz, the eddy current loss will dominate.

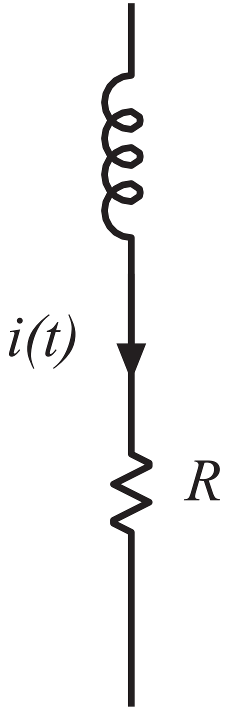
- The core loss directly influences the performance of the devices.

Loss mechanisms in magnetic devices

Core materials

Core type	B_{sat}	Relative core loss	Applications
Laminations iron, silicon steel	1.5 - 2.0 T	high	50-60 Hz transformers, inductors
Powdered cores powdered iron, molypermalloy	0.6 - 0.8 T	medium	1 kHz transformers, 100 kHz filter inductors
Ferrite Manganese-zinc, Nickel-zinc	0.25 - 0.5 T	low	20 kHz - 1 MHz transformers, ac inductors

Loss mechanisms in magnetic devices



DC resistance of wire

$$R = \text{p}\frac{l_b}{A_w}$$

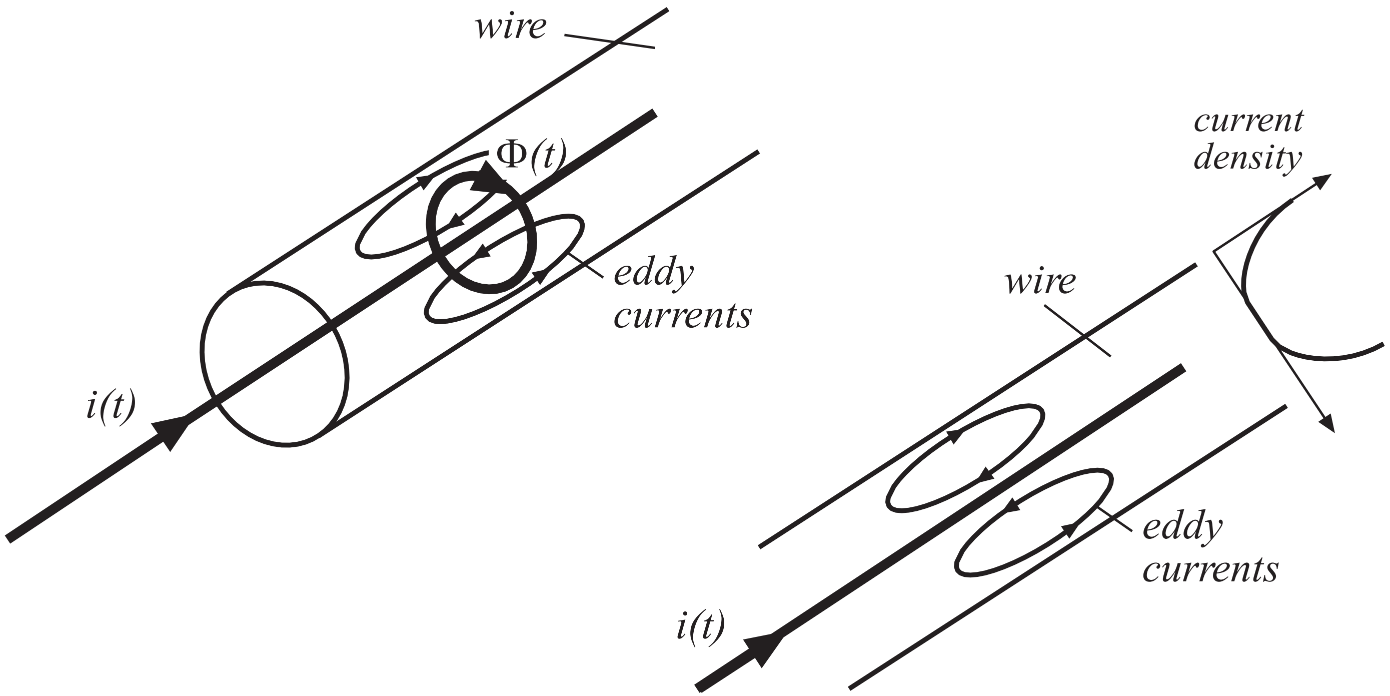
where A_w is the wire's bare cross-sectional area, and l_b is the length of the wire. The resistivity is equal to $1.724{\cdot}10^{-6}\ \Omega$ cm for soft-annealed copper at room temperature. This resistivity increases to $2.3{\cdot}10^{-6}\ \Omega$ cm at 100˚C.

The wire resistance leads to a power loss of

$$P_{cu} = I^2_{rms} R$$

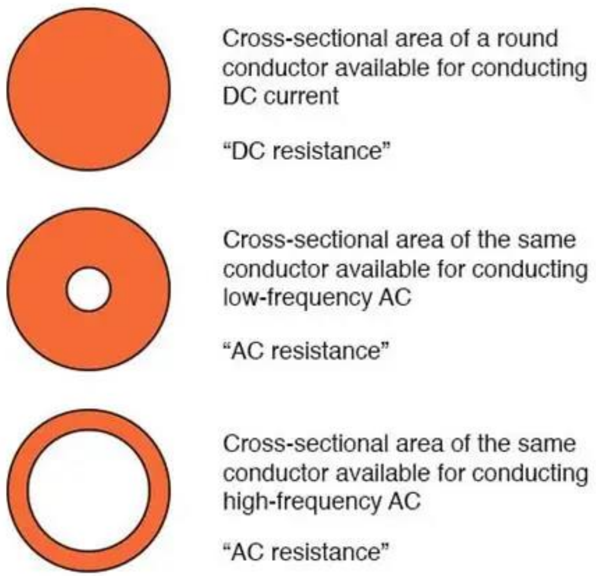
Eddy currents in winding conductors

Introduction to the skin and proximity effects



- Skin effect exists with AC current.

- The eddy currents generated by the main current repel the source, causing the current density to concentrate on the outer side of the conductor.



Eddy currents in winding conductors

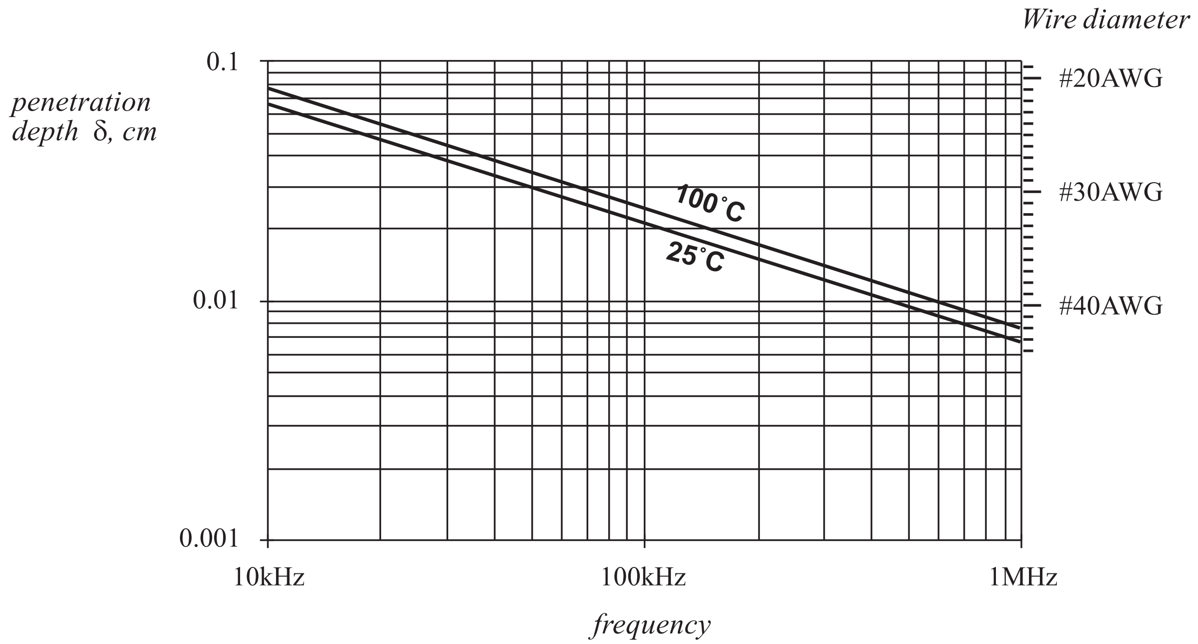
Penetration depth δ

For sinusoidal currents: current density is an exponentially decaying function of distance into the conductor, with characteristic length δ known as the *penetration depth* or *skin depth*.

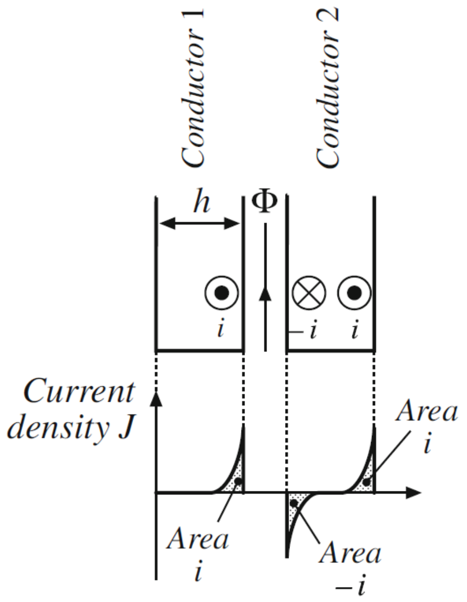
$$\delta = \sqrt{\frac{\rho}{\pi \mu f}}$$

For copper at room temperature:

$$\delta = \frac{7.5}{\sqrt{f}}\,\mathrm{cm}$$

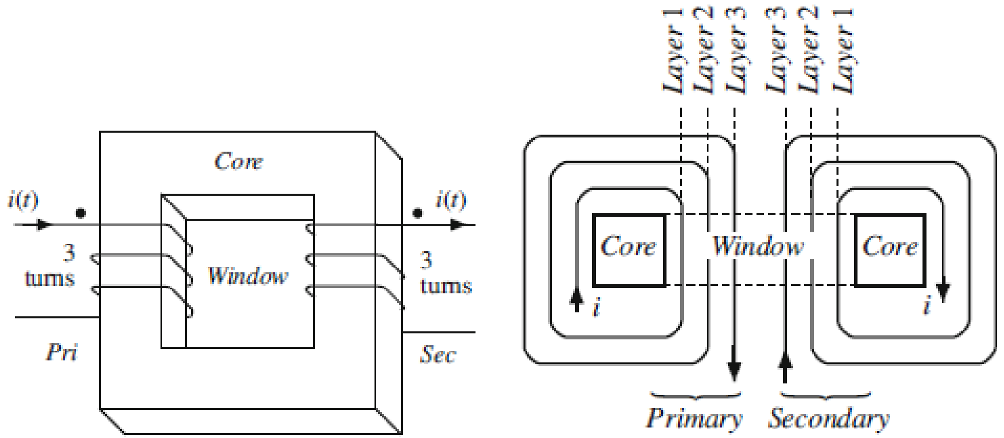


Eddy currents in winding conductors

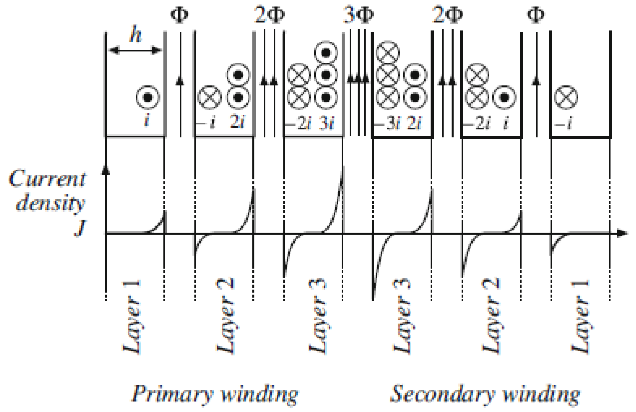


A conductor that carries a high-frequency current *i*(*t*) induces copper loss in an adjacent conductor by a phenomenon known as the ***proximity effect***.

Conductor 1 carries a high-frequency sinusoidal current i(t), whose penetration depth δ is much smaller than the thickness h of conductors 1 or 2. Conductor 2 is open-circuited, so that it carries a net current of zero. However, it is possible for eddy currents to be induced in conductor 2 by the current i(t) flowing in conductor 1.



Effective core geometry and winding geometry



Distribution of currents on surfaces of conductors